บทความวิชาการ

การใช้เลขสุ่มเพื่อจำลองสถานการณ์

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บทคัดย่อ

การนำคณิตศาสตร์มาประยุกต์ใช้เพื่ออธิบายปรากฏการณ์ทั้งหลายที่เกิดขึ้น ซึ่งเป็นที่รู้จักกันในนามตัวแบบเชิงคณิตศาสตร์นั้น มีประโยชน์อย่างยิ่งต่อการนำไปประยุกต์ใช้ในทุกด้าน ถ้าหากตัวแบบเชิงคณิตศาสตร์ที่สร้างขึ้นมานั้นสามารถอธิบายปรากฏการณ์เหล่านั้นได้แม่นยำ การใช้เลขสุ่มเพื่อจำลองสถานการณ์ถือว่าเป็นการสร้างตัวแบบเชิงคณิตศาสตร์อีกแบบหนึ่งที่มีความสำคัญ และประยุกต์ใช้กันอย่างแพร่หลาย บทความนี้มีจุดประสงค์เพื่อนำเสนอการใช้เลขสุ่มเพื่อจำลองสถานการณ์ มีการอธิบายให้เห็นถึงการใช้เลขสุ่มเพื่อจำลองสถานการณ์เชิงกำหนดและสถานการณ์เชิงความน่าจะเป็นที่ง่ายต่อการเข้าใจ มีการสาธิตการประยุกต์การใช้เลขสุ่มเพื่อจำลองสถานการณ์ในชีวิตประจำวันจริงเกี่ยวกับปัญหาสินค้าคงคลัง ผลลัพธ์ที่ได้เป็นอีกหนึ่งแนวทางในการนำคณิตศาสตร์ไปประยุกต์ใช้จริงเพื่อช่วยให้การวางแผนจัดการเกี่ยวกับสถานการณ์ทั้งหลายให้มีประสิทธิภาพมากขึ้น โดยอาศัยข้อมูลที่มาจากตัวแบบเชิงคณิตศาสตร์ที่สร้างขึ้น

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Academic Article

Using random numbers to simulate situations

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Abstract

Applications on mathematics can be used to explain many phenomena in both science and other areas called mathematical modeling. If such mathematical modeling can explain those phenomena well and be accurate, using random numbers to simulate situations is one of the important mathematical models and also widely used. In this paper, we present using random numbers to simulate determined and probability situations that are easy to understand. We apply the methods in real life situations, namely, warehouse problems. The results show that Mathematics is efficiently used in management in many situations by using relevant mathematical models.

Keywords: Random numbers, simulation, determined situations, probability situation, warehouse

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Situation simulation

A situation simulation is a method to explain the trend and behavior of the situation by developing mathematical models to observe and test how well the model is. If the situation simulation with the model works efficiently, we then apply further to make decisions in the right direction or the desired ways. Hence, a situation simulation is one of the mathematical tools that help to plan or manage things and then make the best decisions for the work we are working on (Robinson, 1997; Giordano, Fox and Horton, 2013).

Situation simulations were studied and have been developed by Neuman and Ulam to solve problems or obstacles in wars by using Monte Carlo techniques with other knowledge in mathematics to make more efficiently. In real problems, they could not experiment so they tried to manipulate the situations in smaller scales but very close to the real ones. They tested and studied for the manipulated ones to solve the problems (Metropolis, 1987). In recent days, new technologies and computers can be used to model situations in many branches, namely, in business (Badri, 1999; Merkuryev, Petuhova, Van Landeghem, and Vansteenkiste, 2002; Al-Harkan and Hariga, 2007; Lee and Farahmand, 2010; Davis, Eckhause, Peterson, Pouy, Sigalas-Markham, and Volovoi, 2013). Simulated situations have advantages because they cost less than testing in the real ones.

One of the ways to simulate the situations is to use random numbers which use random samples from the situations. The simulated situation will use samples instead of populations in real problems by using probability distribution for random variables in real situations which is the standard way to choose from random numbers. Data from this random number table obey the distribution populations in real situations. There are ways to find random numbers (L'Ecuyer, 1990; Ižaríková, 2015) and recently, the software plays important role to do so. This work presents random numbers to simulate in both determined and probabilistic situations. We also demonstrate how to apply random variables in real lives, namely, warehouse problems and these results in the applications of simulated situations.

Simulating Deterministic Behavior: Area Under a Curve

In this section, we explain how to use random numbers to simulate situations in deterministic behaviors that have no probability and time involved. We demonstrate how to find the area of a function. $y = f(x)$ which is continuous and its values: $0 \le f(x) \le M$ on the close interval $a \le x \le b$ for some real number M. The property of the function can be depicted in the figure 5.2. We observe that the area is rectangular with the height M and the length is $b - a$.

A point $P(x, y)$ is randomly chosen in the rectangular by finding two random numbers that are x and y satisfying $a \le x \le b$ and $0 \le y \le M$. This means that such a random number is P on coordinate x and y. After $P(x, y)$ is chosen, if it is under the curve, the random number in y coordinate satisfies $0 \le y \le f(x)$.

We denote A_C the area under the curve,

A_R the area of the rectangular,

P_C the number of points located under the curve,

P the numbers of all random points.

The approximation of the area under the curve can be computed from the following relation

$$\frac{A_C}{A_R} = \frac{P_C}{P}.$$

The above relation is equivalent to the approximation: $A_C \approx \frac{P_C \cdot A_R}{P}$ shown in figure 1.

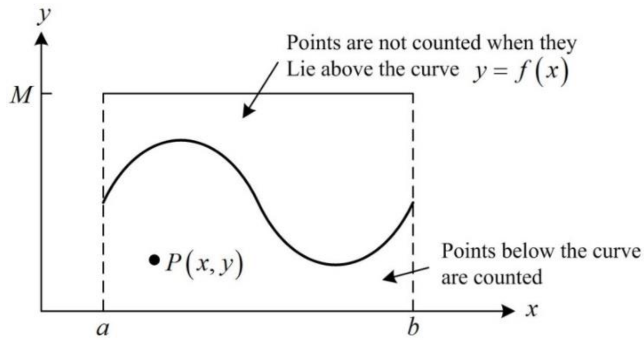

Figure 1. The positive area under $y = f(x)$ on $a \le x \le b$ is contained in the rectangular with the height M and the length $b - a$.

Next, we demonstrate how to use random numbers to find the area under the curve of the function $y = x^2$ on the closed interval $0 \le x \le 1$. A simulated algorithm for finding the area under the curve can be concluded as follows (Applied from Giordano, Fox and Horton, 2013).

Input all random points are n points in this simulation
Output the area is equal to the approximation of the area under the curve $y = x^2$ *on* $0 \le x \le 1$ and then $0 \le x^2 \le 1$
Step 1 start with $counter = 0$
Step 2 for $i = 1,2,\ldots,n$ do Step 3-5
Step 3 compute random coordinates x_i y_i satisfying $0 \le x_i \le 1$ and $0 \le y_i \le 1$
Step 4 compute x_i^2 for random x_i
Step 5 if $y_i \le x_i^2$ then add $counter$ by 1. If not, $counter$ does not change
Step 6 compute $Area \approx \frac{(1)(1-0)counter}{n}$
Step 7 OUTPUT (AREA)
STOP

The result from using Microsoft Excel 2010 can find random numbers to find the area under the curve $y = x^2$, where $0 \le x \le 1$ which has the actual value of $\frac{1}{3} = 0.333\ldots$ as shown in figure 2.

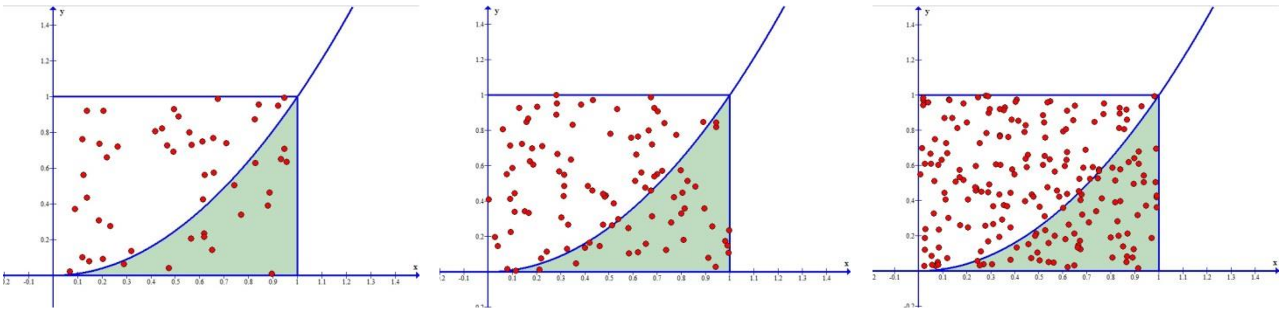

Figure 2. How to use random numbers to simulate situations to find the area under the curve.

From figure 2, the left figure uses 50 random numbers and the approximated area is 0.3. The middle figure uses 100 random numbers and the approximated area is 0.31. The right figure uses 200 random numbers and the approximated area is 0.325. The three figures show the more random numbers increase, the more accurate of the area under the graph is.

Simulating Probabilistic Behavior: Roll of a Fair Dice

To use random numbers for simulating the probabilistic situations, first of all, we must understand the probability. Probability is the subject that studies how the situations randomly happen or how can we measure in numbers of the possibility of the occurrence of situations. For example, the numbers are small, the events rarely happen but the numbers are large, the events or the situations are most likely happen. It can be viewed as long term average, for example, if the time is long, the probability can be measured in ratio.

We denote n_E the numbers of events or situations we are interested in,

n_S the numbers of all events.

The probability of interested events is n_E / n_S.

This topic aims to demonstrate how random numbers can be used to simulate the probabilistic behaviors by considering rolling a fair dice.

Rolling a fair dice, we consider six events because the labels of the dice are $\{1,2,3,4,5,6\}$ and the probability of each score is equally which is $1/6$.

We denote n_D the number of elements in the set $\{1,2,3,4,5,6\}$,

n_T the numbers of all trials.

The probability of each score is n_D / n_T.

In rolling the dice, we will not do it for real, but we will use random numbers instead to simulate the situation as shown by the following algorithm (Applied from Giordano, Fox and Horton, 2013).

Input The all random numbers n in simulation
Output The probability of occurring scores $\{1,2,3,4,5,6\}$
Step 1 *Start* $counter1$ *to* $counter6$ *is* 0
Step 2 For $i = 1,2,\ldots,n$ do Step 3-4
Step 3 Randomly choose $0 \le x_i \le 1$
Step 4 If x_i is in this interval, we add $counter$ properly
$0 \le x_i \le 1/6 \quad counter1 = counter1 + 1$
$1/6 < x_i \le 2/6 \quad counter2 = counter2 + 1$
$2/6 < x_i \le 3/6 \quad counter3 = counter3 + 1$
$3/6 < x_i \le 4/6 \quad counter4 = counter4 + 1$
$4/6 < x_i \le 5/6 \quad counter5 = counter5 + 1$
$5/6 < x_i \le 1 \quad counter1 = counter1 + 1$
Step 5 The probability of occurring scores $j = \{1,2,3,4,5,6\}$ is $counter\ j / n$
Step 6 Output *the probability*
STOP

The result by using Microsoft Excel 2010 to use random numbers for simulating the probability of occurring scores for rolling a fair dice can be concluded as follows:

Scores	Probability for simulating			Theoretical probability
	100 times	1,000 times	10,000 times	
1	0.180	0.159	0.167	0.617
2	0.160	0.173	0.176	0.617
3	0.130	0.187	0.164	0.617
4	0.130	0.155	0.159	0.617
5	0.140	0.182	0.169	0.617
6	0.260	0.144	0.165	0.617

The results by using random numbers in rolling dice show that the more we have trails, the more accuracy of the probability close to the theoretical ones.

Simulating Inventory Problems: Ice Demand Quantity

In this example, we present application of using random numbers to simulate the situations in real lives in warehouse problems and then the result can point out clearly how to apply the application of mathematics. The situation is to simulate the quantity of ice in the restaurant daily. Ice is not easy to keep it and we try to use wisely to save money so for this simulation, we collect data as the following table.

4	8	4	13	11	7	4	11	3	11
5	12	11	3	5	12	6	11	13	8
5	12	3	9	5	5	6	11	12	12
3	12	10	7	10	3	17	6	12	14
6	5	14	4	13	13	5	3	14	13
5	9	17	6	17	8	13	2	15	15

From the data, we find the frequency distribution, frequency, relative frequency, cumulative relative frequency and determine interval random numbers:

Ice quantity (unit : bag)	frequency	Relative frequency	Cumulative relative frequency	Interval random numbers
2-5	19	0.32	0.32	$0 \le x \le 0.32$
6-9	12	0.20	0.52	$0.32 < x \le 0.52$
10-13	21	0.35	0.87	$0.52 < x \le 0.87$
14-17	8	0.13	1.00	$0.87 < x \le 1$

Determine the median in each interval for the quantity of ice q satisfying cumulative relative frequency to find interval random numbers x which give the corresponding values in the below table:

x	0	0.32	0.52	0.87	1
q	2	3.5	7.5	11.5	15.5

The linear equation below explains to find the ice quantity when q and random numbers x on the intervals are given:

Interval of random numbers	Ice quantity
$0 \le x \le 0.32$	$q = 4.6875x + 2$
$0.32 < x \le 0.52$	$q = 20x - 2.9$
$0.52 < x \le 0.87$	$q = 11.4286x + 1.5571$
$0.87 < x \le 1$	$q = 30.7692x - 15.2692$

From the equations, we find the graphs for using random numbers with warehouse problems which here are the ice quantity as shown the figure 3:

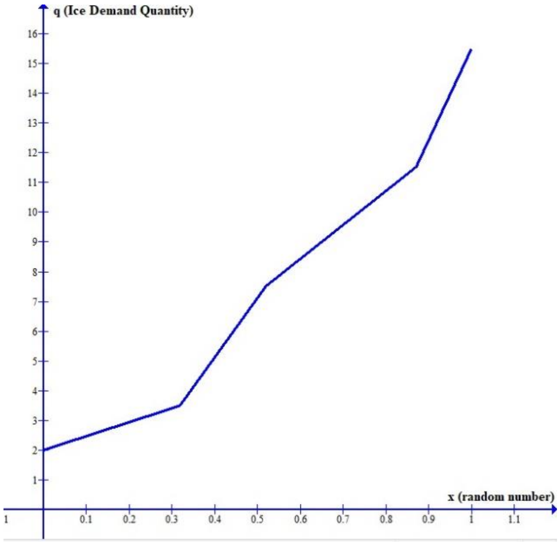

Figure 3. Graph for using random numbers with warehouse problems in the case of using ice. If wanting to know the ice quantity daily, we can find the random numbers and then consider how much those numbers give the ice quantity. The simulated ice quantity is the desired ice quantity daily.

From the graph for using random numbers with warehouse problems in the case of using ice, If wanting to know the ice quantity daily, we can find the random numbers and then consider how much those numbers give the ice quantity. The simulated ice quantity is the desired ice quantity daily. This process can conclude in an algorithm to explain random numbers in warehouse problems with ice quantity (Applied from Giordano, Fox and Horton, 2013).

Input Total random number n in simulation process

Output Ice quantity q_i

Step 1 For $i = 1, 2, \ldots, n$ do Step 2-3

Step 2 Find random numbers $0 \le x_i \le 1$

Step 3 If x_i is in this interval, then add and calculate q_i properly

if $0 \le x_i \le 0.32$ then $q_i = 4.6875x_i + 2$

if $0.32 < x_i \le 0.52$ then $q_i = 20x_i - 2.9$

if $0.52 < x_i \le 0.87$ then $q_i = 11.4286x_i + 1.5571$

if $0.87 < x_i \le 1$ then $q_i = 30.7692x_i - 15.2692$

Step 4 Output print q_i for $i = 1, 2, \ldots, n$

STOP

We will simulate the situation with ice quantity in five days, the results using Microsoft Excel 2010 can be shown on the below table:

Random number	Ice quantity from the simulation by using random numbers (unit: bags)
0.55	8
0.62	9
0.08	2
0.26	3
0.71	10

In the application point of view, we can use results from simulation directly which is preparing ice for five days using the quantity in bags 8, 9, 2, 3, and 10, respectively or we can simulate the situation more than five days to find averages for preparing ice.

Summary

Using random numbers to simulate situations is one of the mathematical applications to see and resolve the problems in real lives or problems with large scales that cannot be evaluated directly. If results from simulations work properly meaning that they are closed to real situations and they can be applied to study further behaviors of the real ones. This leads to efficiently simulate more sophisticated problems. This work has presented using random numbers in both deterministic and probabilistic situations. Moreover, we demonstrate how to simulate the situations in real life to study and resolve real problems. The results provide great advantages for further or more complicated problems.

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