บทความวิจัย

การสร้างตัวแบบเชิงคณิตศาสตร์เกี่ยวกับสถานะตลาดหุ้นโดยใช้ระบบสมการเชิงผลต่าง

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บทคัดย่อ

การวิจัยครั้งนี้มีวัตถุประสงค์เพื่อนำเสนอการสร้างตัวแบบเชิงคณิตศาสตร์ของสถานะตลาดหุ้นโดยใช้ระบบสมการเชิงผลต่าง เริ่มต้นด้วยการศึกษารูปแบบการวิเคราะห์ในรูปลูกโซ่มาร์คอฟแล้วแปลงเป็นระบบสมการเชิงผลต่าง ซึ่งสามารถพิจารณาผลเฉลยเชิงวิเคราะห์ แตกต่างจากการหาผลเฉลยตัวแบบที่อยู่ในรูปลูกโซ่มาร์คอฟ ที่ต้องอาศัยวิธีการเชิงตัวเลขเพื่อหาผลเฉลยเชิงตัวเลข ในขั้นตอนการพิจารณาผลเฉลยของระบบสมการเชิงผลต่างนั้น มีการใช้โปรแกรมวูลแฟรมแอลฟาเพื่อช่วยหาสมการลักษณะเฉพาะ ค่าเจาะจง และค่าเวกเตอร์เจาะจง และมีการใช้โปรแกรมไมโครซอฟท์เอกซ์เซล ในการหาผลเฉลยเชิงตัวเลข และการแสดงกราฟของผลเฉลย สำหรับตัวแบบเชิงคณิตศาสตร์เกี่ยวกับสถานะหุ้นนั้น มีการพิจารณาสถานะเป็น ตลาดขาขึ้น ตลาดขาลง และตลาดหยุดนิ่ง การวิเคราะห์สถานะเพื่อให้เห็นพฤติกรรมระยะยาว กรณีหุ้นที่นำมาศึกษานี้ มีการพิจารณาค่าเริ่มต้นของสถานะ 3 กรณี คือ [1 0 0], [0 1 0] และ [0 0 1] ซึ่งผลการศึกษาพบว่าพฤติกรรมระยะยาวของสถานะหุ้นที่มีค่าเริ่มต้นทั้ง 3 กรณีได้ผลลัพธ์เหมือนกัน คือ มีความน่าจะเป็นของสถานะตลาดขาขึ้นเป็น 0.6250 ตลาดขาลงเป็น 0.3125 และตลาดหยุดนิ่ง 0.0625 ชี้ให้เห็นว่าเมื่อเวลาผ่านไปหุ้นที่นำมาศึกษามีความน่าจะเป็นที่จะเป็นตลาดขาขึ้น ซึ่งจะเป็นข้อมูลสารสนเทศเชิงคณิตศาสตร์ที่มีประโยชน์ข้อมูลหนึ่งเพื่อใช้พิจารณาร่วมกับข้อมูลด้านอื่นในการซื้อขาย

คำสำคัญ: ตัวแบบเชิงคณิตศาสตร์ ระบบสมการเชิงผลต่าง สถานะตลาดหุ้น

อ้างอิงบทความนี้

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Research Article

Mathematical modeling of stock market states using the system of a difference equation

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Abstract

The objective of this research is to present the mathematical model of stock market states using the system of a difference equation. Beginning with the study of the analysis model in the Markov chain and converting it into a system of difference equations, which can be considered analytical solution different from finding the solution in the form of a Markov chain that requires a numerical method to find the numerical solution. In the process of determining the solution of the system of a difference equation the Wolfram Alpha program is used to help for finding the characteristic equation, eigenvalues, and eigenvectors and the use of Microsoft Excel to find the numerical solution and show the solution graph. A mathematical model about the stock market states is considered bull markets, bear markets and stagnant markets. Analysis of long-term behavior in the case of stocks studied in this the initial state of 3 cases are considered, which are [1 0 0], [0 1 0] , and [0 0 1]. The results of the study show that the long-term behavior of stock states initial with all 3 cases yields results. Likewise, there is a probability of a bull market status of 0.6250, a bear market of 0.3125 , and a stagnant market of 0.0625, suggesting that over time the stock studied has a probability of an uptrend, which will be useful mathematical information to be considered together with other trading data.

Keywords: mathematical models, system of difference equations, stock market states

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ลิขสิทธิ์โดย คณะวิทยาศาสตร์ มหาวิทยาลัยอุบลราชธานี

Introduction

The Markov chain is a mathematical tool which applied the theory of probability by studying the behavior or the probability of operating a situation that considering past and future situations are independent of each other. When we are aware of the present situation, that is, if the present situation is known, then no further information is needed about past situations to predict the future. This process will be allowed for even more reductions in the amount of information and be possible to study or predict the scenarios in the future (Andrieu, Doucet and Holenstein, 2010; Brooks, Gelman and et al, 2011; Bidabad and Bidabad, 2019; Yu and Sato, 2019).

Another from the study of mathematical modeling, It is the conversion of real problems to problems in the form of mathematical equations that demonstrate the role and benefits of mathematical models in solving problems, planning, or predicting future events (Fox, 2011; Giordano, Fox ,and Horton, 2013; Albright and Fox, 2019).

This article presents a study and analysis of the state of the stock market in the form of a Markov chain model. The model makes it known for changes in the stock market status from a state to a state (Zhang and Zhang, 2009; Vasanthi, Subha and Nambi, 2011; Svoboda and Lukas, 2012; Myers, Wallin and Wikström, 2017). However, it was found that the analysis of stock status in mathematical models with the Markov chain is less convenient to study interest cases and consider long-term behavior because it is not in the analytical solution form.

For this reason, the stock status models analyzed in the Markov chains are converted to the system of difference equations, which can be considered in analytical solutions and consequenctly, described the mathematical model studies (Fox, 2011; Giordano, Fox and Horton, 2013; Albright and Fox, 2019). It is different from considering the solution to the model in the Markov chain that requires a numerical method to find numerical solutions, which cannot immediately yield the results of the solutions of interest by substituting variables. The stock that will be studied here has 3 initial conditions : [1 0 0], [0 1 0] and [0 0 1] in order to describe the long-term behaviors of the stocks being considered as probability in each position and, furthermore, being used for future trading planning decisions.

System of linear difference equations for three variables

Consider the solution of the system of a difference equation of 3 variables from the state form represented by the Markov chain as shown.

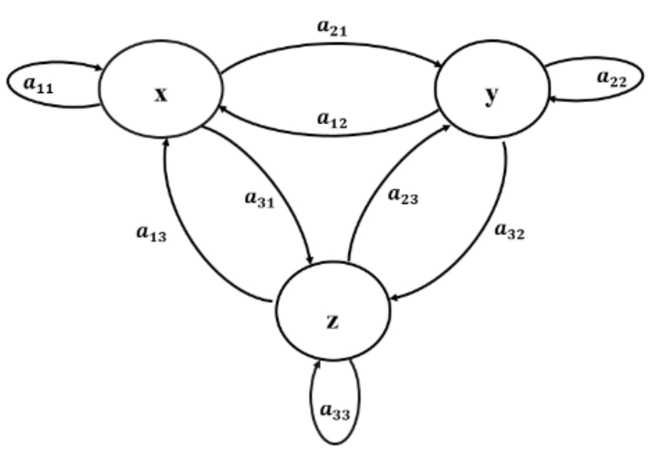

Figure 1. Markov chain containing 3 states.

Markov Chain containing 3 states will be converted into the system of a linear difference equation of 3 variables as follows.

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$$x_{n+1} = a_{11}x_n + a_{12}y_n + a_{13}z_n$$
$$y_{n+1} = a_{21}x_n + a_{22}y_n + a_{23}z_n \qquad (1)$$
$$z_{n+1} = a_{31}x_n + a_{32}y_n + a_{33}z_n$$

It can be converted into a matrix

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & {}_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix}$$

Set to

$$x_n = a\lambda^n, y_n = b\lambda^n, z_n = c\lambda^n$$

From (1), so that

$$\begin{bmatrix} a\lambda^{n+1} \\ b\lambda^{n+1} \\ c\lambda^{n+1} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & {}_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} a\lambda^n \\ b\lambda^n \\ c\lambda^n \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \lambda^{n+1} = \begin{bmatrix} a_{11} & a_{12} & {}_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \lambda^n$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \lambda = \begin{bmatrix} a_{11} & a_{12} & {}_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \qquad (2)$$

Let

$$v = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \text{ and } M = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

From (2) so that

$$M\bar{v} = \lambda\bar{v}$$
$$(M - I\lambda)\bar{v} = \bar{0}$$
$$\det(M - \lambda I)\bar{v} = 0$$
$$\begin{vmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{vmatrix} = 0$$
$$[(a_{11}-\lambda)(a_{22}-\lambda)(a_{33}-\lambda) + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}] -$$
$$[a_{31}a_{13}(a_{22}-\lambda) + a_{31}a_{13}(a_{22}-\lambda) + a_{12}a_{21}(a_{33}-\lambda)] = 0$$

Hence the characteristic equation is in the form of a cubic polynomial equation, $a\lambda^3 - b\lambda^2 + c\lambda - d = 0$. The eigenvalues, λ_1, λ_2 ,and λ_3 are substituted into the equation $(M - I\lambda)\bar{v} = 0$ to get the eigenvectors, $\bar{v}_1, \bar{v}_2$, and $\bar{v}_3$ respectively.

Consequenctly, we obtain the following system:

$$\begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix} = \begin{bmatrix} a\lambda^n \\ b\lambda^n \\ c\lambda^n \end{bmatrix}.$$

Therefore, the solution of the system of equation (1) is shown in the following.

$$\begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix} = \overline{v_1}\lambda_1^{n_1} + \overline{v_2}\lambda_2^{n} + \overline{v_3}\lambda_3^{n}.$$

Mathematical modeling of stock market states

Consider the solution of the system of three variables of difference equations from the stock market state model (Myers, Wallin and Wikström, 2017). We asuume that the probability of a change in state is shown in the Figure 2.

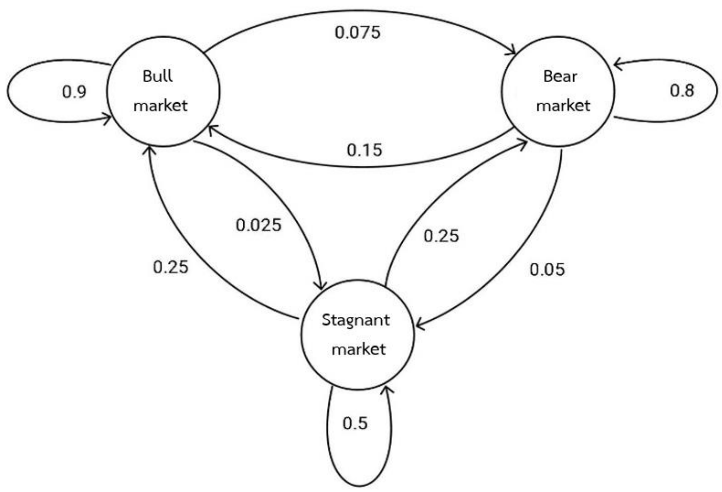

Figure 2. The status of the stock market used in the developed mathematical model.

Let n represent the number of times the stock market position.

I_n represents the increasing state at time n (bull market).

D_n represents the decreasing state at time n (bear market).

S_n represents the standing state at time n (stagnant market).

It can be converted into a system of linear difference equations for three variables as follows

$$\begin{aligned} I_{n+1} &= 0.9\,I_n + 0.15\,D_n + 0.25S_n \\ D_{n+1} &= 0.075\,I_n + 0.8\,D_n + 0.25\,S_n \\ S_{n+1} &= 0.025\,I_n + 0.05\,D_n + 0.5S_n \end{aligned} \qquad (3)$$

System of equation (3) can be converted into the matrix form as follows.

$$\begin{bmatrix} I_{n+1} \\ D_{n+1} \\ S_{n+1} \end{bmatrix} = \begin{bmatrix} 0.9 & 0.15 & 0.25 \\ 0.075 & 0.8 & 0.25 \\ 0.025 & 0.05 & 0.5 \end{bmatrix} \begin{bmatrix} I_n \\ D_n \\ S_n \end{bmatrix}$$

Let

$$I_n = a\lambda^n, D_n = b\lambda^n, S_n = c\lambda^n$$

Substituting back into the system of equation (3) we get that

$$\begin{bmatrix} a\lambda^{n+1} \\ b\lambda^{n+1} \\ c\lambda^{n+1} \end{bmatrix} = \begin{bmatrix} 0.9 & 0.15 & 0.25 \\ 0.075 & 0.8 & 0.25 \\ 0.025 & 0.05 & 0.5 \end{bmatrix} \begin{bmatrix} a\lambda^n \\ b\lambda^n \\ c\lambda^n \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \lambda^{n+1} = \begin{bmatrix} 0.9 & 0.15 & 0.25 \\ 0.075 & 0.8 & 0.25 \\ 0.025 & 0.05 & 0.5 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \lambda^n$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \lambda = \begin{bmatrix} 0.9 & 0.15 & 0.25 \\ 0.075 & 0.8 & 0.25 \\ 0.025 & 0.05 & 0.5 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad (4)$$

Let

$$v = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \text{ and } M = \begin{bmatrix} 0.9 & 0.15 & 0.25 \\ 0.075 & 0.8 & 0.25 \\ 0.025 & 0.05 & 0.5 \end{bmatrix}$$

From the system of equation (4) we get that

$$M\bar{v} = \lambda \bar{v}$$

$$(M - I\lambda)\bar{v} = \bar{0}$$

$$\det(M - \lambda I)\bar{v} = 0$$

$$\begin{vmatrix} 0.9-\lambda & 0.15 & 0.25 \\ 0.075 & 0.8-\lambda & 0.25 \\ 0.025 & 0.05 & 0.5-\lambda \end{vmatrix} = 0$$

Characteristic equation is $\lambda^3 - 2.2\lambda^2 + 1.54\lambda - 0.34 = 0$ which can find eigen values as follows

$$\lambda_1 = 1$$

$$\lambda_2 = \frac{1}{10}\left(6+\sqrt{2}\right) \approx 0.74142$$

$$\lambda_3 = \frac{1}{10}\left(6-\sqrt{2}\right) \approx 0.45858$$

The eigen vectors corresponding to each eigen values respectively have the following values.

$$v_1 = \begin{bmatrix} 10 \\ 5 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -11.657 \\ 10.657 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -0.34315 \\ -0.65685 \\ 1 \end{bmatrix}$$

So, the solution to the system of equations is

$$\begin{bmatrix} I_n \\ D_n \\ S_n \end{bmatrix} = c_1 \begin{bmatrix} 10 \\ 5 \\ 1 \end{bmatrix} (1)^n + c_2 \begin{bmatrix} -11.657 \\ 10.657 \\ 1 \end{bmatrix} (0.74142)^n + c_3 \begin{bmatrix} -0.34315 \\ -0.65685 \\ 1 \end{bmatrix} (0.45858)^n \quad (5)$$

Long-term behavior of solutions

From the operation of mathematical modeling on the stock market status using a system of differential equations. The solution in each case of the initial value can be determined by substituting the value n which is the amount of time of the stock market position, where $0 \le n < \infty$. The solution can be summarized as follows.

In case of initial value $[1 \quad 0 \quad 0]$, the solution of system of a linear difference equation is

$$I_n = 0.625(1)^n + 0.3642765872(0.74142)^n + 0.01072357476(0.45858)^n$$

$$D_n = 0.3125(1)^n - 0.3330269872(0.74142)^n + 0.02052682524(0.45858)^n$$

$$S_n = 0.0625(1)^n - 0.0312496(0.74142)^n - 0.0312504(0.45858)^n$$

When solutions are considered from a mathematical model using a system of a difference equation in order to predict future stock market status, it was found that in the case of initial value [1 0 0], in the first period which are the 1^{st}-5^{th} week, there would be inconstant values and the graph is stable from the 6^{th}-8^{th} week and afterward as shown in figure 3.

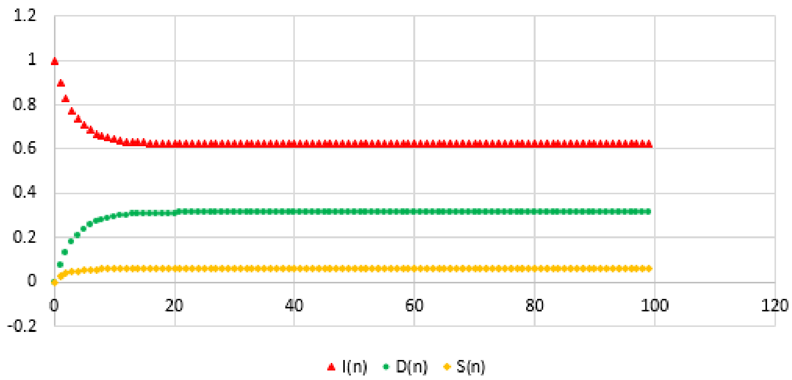

Figure 3. Long-term behavior analysis from solutions in the case of initial value [1 0 0].

In case of initial value $[0 \quad 1 \quad 0]$, the solution of system of a linear difference equation is

$$I_n = 0.625(1)^n - 0.6660541689(0.74142)^n + 0.0410537797(0.45858)^n$$

$$D_n = 0.3125(1)^n + 0.6089164689(0.74142)^n + 0.0785842203(0.45858)^n$$

$$S_n = 0.0625(1)^n + 0.0571377(0.74142)^n - 0.119638(0.45858)^n$$

Similar to the periods of times in the first case, the graph behaviours of solutions to the system are shown in figure 4.

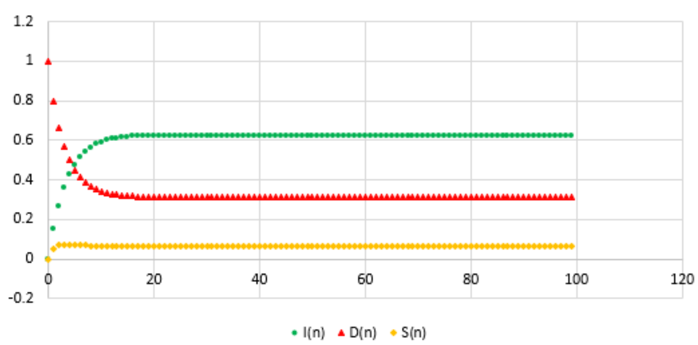

Figure 4. Long-term behavior analysis from solutions in the case of initial value [0 1 0].

In case of initial value $[0 \quad 0 \quad 1]$, the solution of system of a linear difference equation is

$$I_n = 0.625(1)^n - 0.3124950275(0.74142)^n - 0.3125039598(0.45858)^n$$

$$D_n = 0.3125(1)^n + 0.2856875275(0.74142)^n - 0.5981880402(0.45858)^n$$

$$S_n = 0.0625(1)^n + 0.0268076(0.74142)^n + 0.910692(0.45858)^n$$

The solutions of the system in this case are shown in figure 5. The periods of times are similar to both previous cases yet the graphs are different because of the initial condition.

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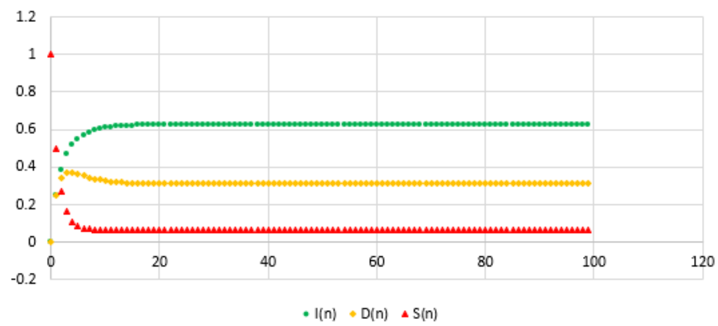

Figure 5. Long-term behavior analysis from solutions in the case of initial value [0 0 1].

Summary

By creating a mathematical model of the stock market status using the system of a difference equation, this makes it possible to consider the solutions in the form of analytical solutions, as well as makes it convenient to apply them to explain the mathematical models studied. It is different from considering the solution to the model in the Markov chain. A numerical method is required in order to find numerical solutions, which cannot immediately yield the results of the solutions of interest by substituting variables. For the case of stocks that are studied in this model, the initial values of the 3 cases were [1 0 0], [0 1 0] and [0 0 1]. The results of the study found that the long-term behavior of the stock positions with the initial values in all 3 cases yielded the same results. There are a probability of an uptrend of 0.6250, a bearish market of 0.3125, and a stagnation of 0.0625, as Figures 2, 3, and 4, indicating that over time the stocks studied were probably an uptrend. From the process of constructing a mathematical model using the system of a difference equation about the stock market status, this will determine the probability of the future stock market situation. Including the risks which are useful to investors or those interested in studying to be considered in the selection of interest stocks. However, other relevant factors should be considered to have the least impact from risk on buying stocks.

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