JSSE JOURNAL OF SCIENCE & SCIENCE EDUCATION Vol. 7 No. 2 (Jul. – Dec. 2024): 398 - 406
Faculty of Science, Ubon Ratchathani University http://doi.org/10.14456/jsse.2024.31

Research Article in Science Education

Developing Mathematical Problem-Solving Skills Using Polya's Steps: A Problem-Based Learning Approach in Combinatorics for Grade 11 Students

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Received <16 July 2024>; Revised <27 August 2024>; Accepted <2 September 2024>

Abstract

This research aims to 1) compare the academic achievement and mathematical problem-solving skills following Polya's steps before and after learning through problem-based learning, and 2) compare the academic achievement and mathematical problem-solving skills following Polya's steps after problem-based learning with the 80% criterion. The sample consisted of 33 Grade 11 students from Ratchasima Wittayalai School in the first semester of the 2022 academic year. The research instruments included problem-based learning lesson plans, an achievement test, and a mathematical problem-solving skills test. Data were analyzed using percentage, mean, standard deviation, and dependent t-test. The results showed that the academic achievement and mathematical problem-solving skills following Polya's steps were significantly higher after learning than before at the 0.01 level, and the academic achievement and mathematical problem-solving skills after problem-based learning were significantly higher than the 80% criterion at the 0.01 level.

Keywords: Problem-based learning, Polya's problem-solving steps, Combinatorics

บทความวิจัยทางวิทยาศาสตร์ศึกษา

การพัฒนาทักษะการแก้ปัญหาทางคณิตศาสตร์ตามขั้นตอนของโพลยา เรื่องหลักการนับเบื้องต้น ด้วยการจัดการเรียนรู้โดยใช้ปัญหาเป็นฐาน ของนักเรียนชั้นมัธยมศึกษาปีที่ 5

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บทคัดย่อ

การวิจัยครั้งนี้มีวัตถุประสงค์เพื่อ 1) เปรียบเทียบผลสัมฤทธิ์ทางการเรียนและทักษะการแก้ปัญหาทางคณิตศาสตร์ตามขั้นตอนของโพลยาก่อนเรียนและหลังเรียนด้วยการจัดการเรียนรู้โดยใช้ปัญหาเป็นฐาน 2) เปรียบเทียบผลสัมฤทธิ์ทางการเรียนและทักษะการแก้ปัญหาทางคณิตศาสตร์ตามขั้นตอนของโพลยาหลังการจัดการเรียนรู้โดยใช้ปัญหาเป็นฐานกับเกณฑ์ร้อยละ 80 กลุ่มตัวอย่างเป็นนักเรียนชั้นมัธยมศึกษาปีที่ 5 โรงเรียนราชสีมาวิทยาลัย ภาคเรียนที่ 1 ปีการศึกษา 2565 จำนวน 33 คน เครื่องมือที่ใช้ในการวิจัย ได้แก่ แผนการจัดการเรียนรู้โดยใช้ปัญหาเป็นฐาน แบบทดสอบวัดผลสัมฤทธิ์ทางการเรียน และแบบทดสอบวัดทักษะการแก้ปัญหาทางคณิตศาสตร์ วิเคราะห์ข้อมูลโดยใช้ค่าร้อยละ ค่าเฉลี่ย ค่าส่วนเบี่ยงเบนมาตรฐาน และสถิติทดสอบ dependent t-test ผลการวิจัยพบว่า ผลสัมฤทธิ์ทางการเรียนและทักษะการแก้ปัญหาทางคณิตศาสตร์ตามขั้นตอนของโพลยาหลังเรียนสูงกว่าก่อนเรียนอย่างมีนัยสำคัญทางสถิติที่ระดับ .01 และผลสัมฤทธิ์ทางการเรียนและทักษะการแก้ปัญหาทางคณิตศาสตร์หลังการจัดการเรียนรู้โดยใช้ปัญหาเป็นฐานสูงกว่าเกณฑ์ร้อยละ 80 อย่างมีนัยสำคัญทางสถิติที่ระดับ .01

คำสำคัญ: การจัดการเรียนรู้โดยใช้ปัญหาเป็นฐาน ทักษะการแก้ปัญหาทางคณิตศาสตร์ตามขั้นตอนของโพลยา หลักการนับเบื้องต้น

Introduction

The development of human thinking to be rational, creative, and systematically organized is crucial for analyzing and planning solutions to everyday problems. Mathematics plays a significant role in this development and serves as a fundamental tool in the study of science, technology, and other disciplines. The primary goal of mathematics education is to teach students to think critically and develop problem-solving skills applicable to daily life (The Ministry of Education, 2008). Thus, mathematics instruction must emphasize systematic problem-solving methods to develop essential skills for tackling problems and accumulating problem-solving experience based on individual capabilities.

The goal of mathematics education is to develop students into systematic thinkers and problem-solvers. However, assessments like the Program for International Student Assessment (PISA 2009-2015) show that Thai students' mathematics scores are below average and declining (Institute for the Promotion of Teaching Science and Technology, 2013). Observations indicate that Thai students lack the ability to apply mathematical knowledge and skills to real-life situations. Essential mathematical skills include problem-solving, reasoning, connecting concepts, communication, and creativity. Additionally, attitudes towards mathematics and teaching methods are critical. current teaching approach often focuses on direct instruction using problems that do not connect to real-life applications. This causes students to overlook the significance of mathematics and fail to recognize its relevance to daily life. When students are not exposed to problems that resemble real-life situations, they lack the skills to analyze problems, plan solutions, and apply the knowledge they have learned. Consequently, students struggle to use mathematical knowledge to solve complex problems in real-world contexts. Moreover, the emphasis on teaching methods that limit creativity and reasoning restricts the development of critical thinking and the ability to generate new approaches to problem-solving. Therefore, it is essential to promote and develop students "mathematical problem-solving skills" by focusing on connecting knowledge to real-life applications, fostering systematic thinking, and providing opportunities for students to engage with more challenging and relevant problems. (Ransungneon, 2012).

Polya's problem-solving process, widely accepted in education, involves four steps that promote systematic and clear problem-solving: 1) Understanding the problem: students identify what is required and the relationships between given elements; 2) Planning the solution: students find connections and integrate them to form a plan; 3) Executing the plan: students follow the plan or devise new strategies; 4) Reviewing the results: students check if the results are reasonable (Polya, 1957, cited in Suwan, 2011). Using problem-based learning (PBL) is another effective method that focuses on student-centered learning, promoting critical thinking, problem-solving, and meaningful learning. PBL uses problems to motivate and engage students, fostering active learning and the integration of knowledge for practical application.

Given the importance and research findings, as a mathematics teacher, I am interested in studying and developing mathematical problem-solving skills using Polya’s steps through problem-based learning. This approach will benefit teachers in aligning teaching activities with the Basic Education Core Curriculum B.E. 2551 (revised B.E. 2560) and provide a framework for enhancing learning outcomes and problem-solving skills, thereby fostering higher-order thinking and creative processes in students.

Research Objectives

1. To compare the academic achievement and mathematical problem-solving skills following Polya's steps before and after learning through problem-based learning.

2. To compare the academic achievement and mathematical problem-solving skills following Polya's steps after problem-based learning with the 80% criterion.

Conceptual Framework

The researcher studied theories on problem-based learning and relevant research, including works by Khammanee (2011); Stepien and Gallagher (1993), cited in Laolearndee (2011), The Office of the Secretary-General of the Education Council (2017). The research framework can be illustrated as shown in Figure 1.

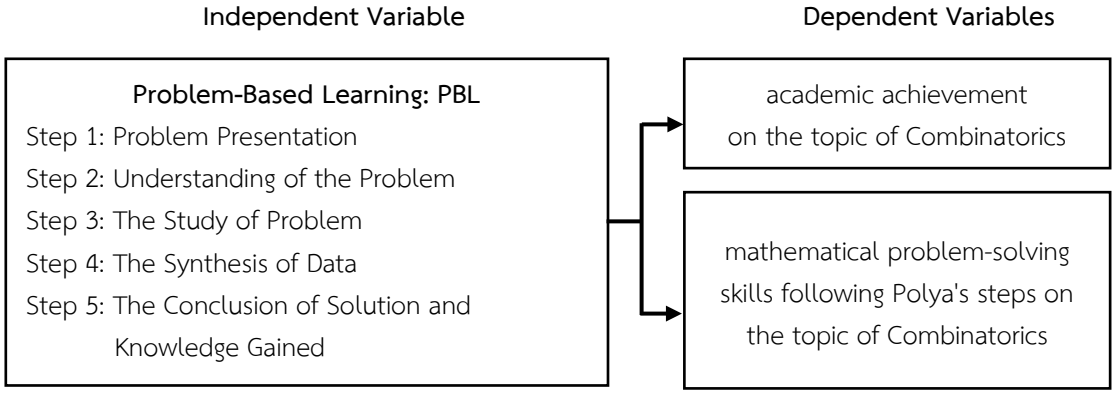

Figure 1. Conceptual Framework

Research Methodology

1. Research Hypotheses

1.1 The academic achievement and mathematical problem-solving skills of Grade 11 students on the topic of combinatorics will be higher after learning through problem-based learning (PBL) compared to before.

1.2 The academic achievement and mathematical problem-solving skills of Grade 11 students, when solving mathematical problems following Polya's steps using problem-based learning (PBL), surpass the 80% benchmark.

2. Population and Sample Group

The population consisted of Grade 11 students in the first semester of the 2022 academic year at Ratchasima Wittayalai School, totaling 685 students across 18 classrooms. The sample group was obtained through cluster sampling, selecting one classroom with 33 students.

3. Research Instruments

3.1 Problem-Based Learning Lesson Plans: A total of 18 lesson plans, each lasting 1 hour, totaling 18 hours. The quality of the lesson plans was evaluated by 5 experts to determine the Index of Item-Objective Congruence (IOC). The IOC was found to be 1.00, indicating that the lesson plans were highly appropriate and aligned with the learning objectives.

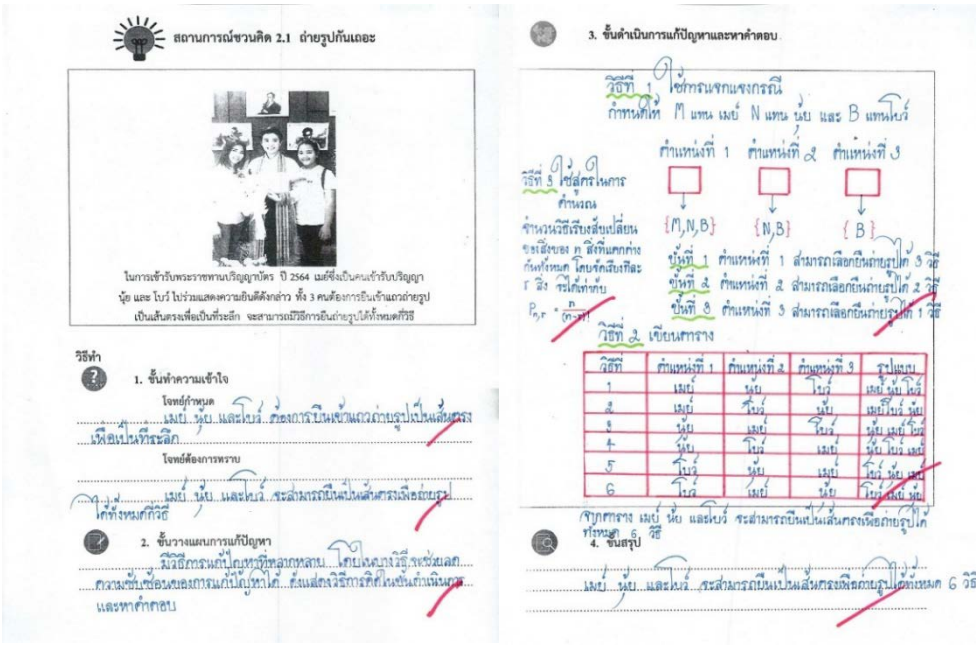

Figure 2. Example of a learning activity following Polya's steps using problem-based learning

3.2 Achievement Test: A multiple-choice test with 5 options, consisting of 30 items. The test was presented to experts for quality evaluation. The test's item congruence was determined, with an acceptable congruence index of at least 0.50 (Nilaphan, 2012). The achievement test was piloted with 30 Grade 11 students, who were not part of the sample group, to analyze item difficulty (p), which ranged from 0.37 to 0.73, and item discrimination (r), which ranged from 0.40 to 0.87. The reliability of the test was calculated using the KR-20 formula, yielding a reliability coefficient of 0.92.

3.3 Mathematical Problem-Solving Skills Test Based on Polya's Steps: An essay test consisting of 5 items. The test was presented to experts for quality evaluation. The test's item congruence was determined, with an acceptable congruence index of at least 0.50 (Nilaphan, 2012). The problem-solving skills test was piloted with 30 Grade 11 students, who were not part of the sample group, from a total of 10 items, selecting 5 items for analysis. The item difficulty (p) ranged from 0.55 to 0.73, and the item discrimination (r) ranged from 0.23 to 0.38. The reliability of the test was calculated using the KR-20 formula, yielding a reliability coefficient of 0.78.

4. Data Collection Procedure

The researcher conducted a problem-based learning (PBL) approach with the following details:

4.1 Pre-test: The sample group of students were given a pre-test, which included a 30-item multiple-choice achievement test and a 5-item essay test based on Polya's problem-solving steps. The scores from these tests were recorded.

4.2 Implementation of PBL Activities: The experimental phase involved 16 hours of PBL activities, following these steps:

Step 1: Problem Presentation

Step 2: Understanding of the Problem

Step 3: Study of the Problem

Step 4: Synthesis of Data

Step 5: Conclusion of the Solution

During the PBL sessions, student performance was recorded each hour using a learning outcome recording form.

Figure 3. 5 Steps of Problem-Based Learning management, as cited in Khammanee (2017).

4.3 Post-test: After completing the PBL activities, the sample group of students were given a post-test, which included the same 30-item multiple-choice achievement test and the 5-item essay test based on Polya's problem-solving steps. The scores from these tests were recorded.

5. Data Analysis

Data analysis using basic statistics includes measures such as percentage, mean, standard deviation and dependent t-test

Results

1. The learning outcomes on Combinatorics of Grade 11 students using problem-based learning as the basis are shown in Table 1.

Table 1. Comparison of the average scores from the pre-test and post-test measuring learning outcomes on Combinatorics before and after implementing problem-based learning:

The learning outcomes	n	mean	S.D.	df	t
pre-test	33	19.94	2.44	32	19.039**
post-test	33	26.02	1.14		

**Significant at the .01 level

From Table 1, the test results measuring the learning achievement on Combinatorics for Grade 11 students showed that post-test scores were significantly higher than pre-test scores at the .01 level of statistical significance with the implementation of problem-based learning.

2. The learning outcomes on Combinatorics of Grade 11 students using problem-based learning compared to benchmark of 80% are shown in Table 2.

Table 2. Comparison of the average scores from the post-test measuring learning outcomes on Combinatorics of Grade 11 students using problem-based learning, with a benchmark of 80%:

Experimental results	n	Full marks	Criterion	mean	S.D.	df	t
post-test	33	30	24	26.02	1.14	32	10.370**

**Significant at the .01 level

From Table 2, the post-test results indicate that the average learning outcomes of Grade 11 students on the topic of Combinatorics, after using problem-based learning (PBL), are significantly higher than the benchmark of 80% (mean = 26.02, S.D. = 1.14). The t-test analysis reveals a statistically significant difference at the .01 level ($t = 10.370$, $p < .01$), confirming that the students' learning outcomes exceed the established criterion.

3. The mathematical problem-solving skills using Polya's method on Combinatorics of Grade 11 students improved significantly after implementing problem-based learning, as shown in Table 3.

Table 3. Comparison of the average scores from the pre-test and post-test measuring problem-solving skills in mathematical counting principles using Polya's method, before and after implementing problem-based learning:

The learning outcomes	n	mean	S.D.	df	t
pre-test	33	27.64	2.81	32	37.172**
post-test	33	45.26	3.23		

**Significant at the .01 level

From Table 3, the post-test scores measuring mathematical problem-solving skills using Polya's method significantly improved after implementing problem-based learning compared to the pre-test scores at the .01 significance level.

4. Mathematical problem-solving skills according to Polya's steps on Combinatorics of Grade 11 students, with a benchmark of 80%, as shown in Table 4.

Table 4. Comparison of the average scores from the post-test measuring mathematical problem-solving skills using Polya's method on Combinatorics, with a benchmark of 80%:

Experimental results	n	Full marks	Criterion	mean	S.D.	df	t
post-test	33	50	40	45.26	3.23	32	9.500**

**Significant at the .01 level

From Table 4, The post-test results show that the average mathematical problem-solving skills of Grade 11 students using Polya's method on the topic of combinatorics are significantly higher than the benchmark of 80% (mean = 45.26, S.D. = 3.23). The t-test analysis indicates a statistically significant difference at the .01 level ($t = 9.500$, $df = 32$), confirming that the students' mathematical problem-solving skills exceed the established criterion.

Discussion

1. Academic achievement: The academic performance regarding basic counting principles through problem-based learning (PBL) showed post-learning scores higher than pre-learning and above the 80% criterion. It clearly indicates fundamental problem-solving skills following Polya's steps and PBL, emphasizing self-constructed knowledge from real problem situations. Students start by solving problems, gather ideas, connect knowledge, sequence steps to find answers, and stimulate diverse and

accurate problem-solving. This is consistent with Thanwiset and Keeratichamroen (2018). It was found that post-PBL academic performance was statistically significantly higher than pre-PBL performance and exceeded the 70% threshold ($p < .05$). Correspondingly, Abdullah, Tarmizi and Abu (2010) conducted research to find that mathematical problem-solving skills averaged 83.03%, exceeding the 70% benchmark.

2. Mathematical problem-solving skills: The mathematical problem-solving skills of grade 11 students, using Polya's method, exceeded the 80% criterion post-PBL. This improvement stems from initiating PBL with problem scenarios, where teachers set goals to guide students in problem-solving from given situations. These problems are explained as scenarios or events encountered in daily life, explicable with theory and practice. When presented, the problems serve as starting points in student learning processes, where students explain situations through group processes to maximize learning outcomes. This aligns with Cotic and Zuljan (2009), who selected a sample group of 179 nine-year-old students in Slovenia. Research results indicate that students taught using PBL successfully solved complex mathematical problems more effectively than those taught conventionally. Additionally, it was hypothesized that most Slovenian students scored lower in mathematics problem-solving than students from other countries, despite having good computational skills. The research also showed that PBL teaching methods instilled confidence in students that difficult problem tasks can be solved, aligning with Ounpomma and Pavapuyanon (2017) development of mathematical learning activities based on constructivist theory, focusing on Polya's problem-solving probability. In fifth grade, the academic performance score averaged 15.29, equivalent to 76.45%, with 43 students passing the threshold out of a total of 51 students, equating to 84.313%. Students' problem-solving skills in PBL, focused on Polya's problem-solving theory, averaged 75.70%, reflecting a high level of proficiency.

Recommendations

1. Implementation Suggestions:

1.1 Detailed Problem-solving Steps: Teachers should thoroughly and clearly explain each step of the mathematical problem-solving process to ensure accuracy and understanding. They should also encourage students to engage closely in learning activities to achieve the intended learning objectives.

1.2 Diverse Problem Scenarios: While using different content-specific problem scenarios, teachers should aim for similar approaches to finding solutions. This approach promotes diverse learning experiences and ensures that students develop comprehensive problem-solving skills. It's crucial for teachers to prioritize creating foundational problem scenarios that stimulate students to apply their knowledge and skills fully during problem-solving discussions.

2. Future Research Recommendations:

For future research, it is advisable to explore other innovative learning methods alongside PBL. These could include integrated learning activities, project-based learning combined with inquiry-based learning activities, or learning approaches that encourage knowledge discovery. This would enrich educational practices and potentially enhance students' understanding of the subject matter.

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