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Research Article in Science Education

Python Simulation Activities for Estimating the Area Between Curves for Secondary School Students

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Abstract

This Python simulation learning activity for estimating the area between curves was created to teach students how to simulate problems with deterministic behavior, which is another important step in developing students' simulation modeling competence. The target group of twenty-eight grade 12 students learned to use the Python program to estimate the area between curves using a developed hands-on simulation. After the activity was completed, the students' competency in simulation for estimating the area between curves using Python was evaluated. The results of the evaluation indicate that the designed activity can improve the desired performance. In addition, the evaluation results revealed that students' enjoyment, value, interest, and self-efficacy in performing activities led to positive outcomes. The findings of this study point to a strategy that interested teachers can use to improve students' basic competency and knowledge for learning about simulation modelling further.

Keywords: Python program, Mathematical modelling, Simulation modelling, Simulating deterministic behavior, Area between curves

บทความวิจัยทางวิทยาศาสตร์ศึกษา

กิจกรรมจำลองด้วยโปรแกรมไพทอนเพื่อประมาณพื้นที่ระหว่างกราฟ สำหรับนักเรียนระดับชั้นมัธยมศึกษา

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บทคัดย่อ

กิจกรรมการเรียนรู้ด้วยการจำลองด้วยไพทอน (Python) สำหรับการประมาณค่าพื้นที่ระหว่างเส้นโค้งนี้ถูกออกแบบขึ้นเพื่อสอนนักเรียนเกี่ยวกับการจำลองปัญหาที่มีพฤติกรรมเชิงกำหนด ซึ่งเป็นขั้นตอนสำคัญในการพัฒนาความสามารถในการสร้างแบบจำลองของนักเรียน กลุ่มเป้าหมายคือนักเรียนชั้นมัธยมศึกษาปีที่ 6 จำนวน 28 คน ได้เรียนรู้การใช้โปรแกรมไพทอนเพื่อประมาณค่าพื้นที่ระหว่างเส้นโค้งโดยใช้การจำลองแบบลงมือปฏิบัติที่พัฒนาขึ้น เมื่อเสร็จสิ้นกิจกรรม ได้มีการประเมินความสามารถของนักเรียนในการใช้ไพทอน (Python) เพื่อจำลองและประมาณค่าพื้นที่ระหว่างเส้นโค้ง ผลการประเมินแสดงให้เห็นว่ากิจกรรมที่ออกแบบสามารถปรับปรุงผลการปฏิบัติงานที่ต้องการได้ นอกจากนี้ ผลการประเมินยังแสดงให้เห็นว่านักเรียนมีความสนุกสนาน มองเห็นคุณค่า มีความสนใจ และมีความเชื่อมั่นในตนเองในการทำกิจกรรม ซึ่งนำไปสู่ผลลัพธ์ในเชิงบวก การศึกษานี้ชี้ให้เห็นถึงกลยุทธ์ที่ครูสามารถนำไปใช้เพื่อพัฒนาความสามารถพื้นฐานและความรู้ของนักเรียนในการเรียนรู้เกี่ยวกับการสร้างแบบจำลองต่อไป

คำสำคัญ: โปรแกรมไพทอน ตัวแบบทางคณิตศาสตร์ ตัวแบบของการจำลอง พฤติกรรมการจำลองเชิงกำหนด พื้นที่ระหว่างเส้นโค้ง

Introduction

Mathematical modeling is one way in which mathematics can be used to solve real-world problems effectively. Mathematical modeling consists of four main processes: converting real-world problem situations to mathematics; selecting a method that corresponds to the mathematical model obtained from the problem for the results; converting the results back to describe the actual problem situation at the source; and refining the mathematical model based on the applied model results (Blum and Leiß, 2006; Galbraith, Holton and Turner, 2020; Hartmann, Krawitz and Schukajlow, 2021; Schukajlow, Krug and Rakoczy 2015; Wake, 2015). The development of mathematical modeling competencies has occurred in many educational curricula in response to the need to develop human resources in the country who can effectively apply mathematics to everyday life. A general goal aspect is the development of mathematical modeling competency on basic mathematics content according to curriculum standards for students to use in daily life (English, 2012; Kazak, Pratt and Gökce, 2018; Krutikhina *et al*., 2018; Patel and Pfannkuch, 2018; Shahbari and Peled, 2017; Stillman, 2010). Another intriguing and equally important aspect is the development of unique modeling competencies to encourage students with special interests to pursue higher education in order to become experts in a specific type of mathematical modeling (Farihah, 2019; Frejd and Ärlebäck, 2017; Greefrath and Siller, 2017; Kotelawala, 2011; Leung, 2013; Ortega and Puig, 2017; Rodríguez Gallegos, 2015).

Another important model that can be applied to situations that cannot be directly analyzed is simulation modeling. With the appropriate program on the computer, simulation modeling simulates the actual behavior of a problem situation. Then, examine the data from simulation modeling and apply the results to explain the actual problem situation. (Kin and Chan, 2011; Maria, 1997; Tobrawa *et al*., 2022; Volosencu and Ryoo, 2022; Zárate Ceballos *et al*., 2021). A problem scenario that necessitates simulation modeling to find an answer can be classified as simulation modeling for deterministic behavior problems at first (Barros, 2007; Chowdhury and Liu, 2014; Papageorgiou and Paskov, 1999). and simulation modeling for probabilistic behavior problems (Paler *et al*., 2013; Cecconi *et al*., 2017; Sánchez and Arroyo, 2019; Gandrud and Williams, 2017; Zöllner *et al*., 2021). It is critical to educate early learners about simulating deterministic behavior problems when teaching simulation modeling in general. The problem of determining area and volume will be raised in the lesson to demonstrate the simulation, which can estimate area and volume and is thought to be a solution for deterministic behavior (Albright and Fox, 2019; Giordano, Fox and Horton, 2013; Gordon and Guilfoos, 2017). However, teachers face the challenge that simulations require a suitable simulation tool, which they must be able to use to simulate the results and graph the results to better comprehend the students. And students can practice simulations of deterministic behavioral problems; the selection of tools for learning management is often quite challenging. Because it must choose a tool that is flexible between facilitating the development of independent commands as needed and supporting a pre-made instruction set as a tool to expedite development, it is required to choose a tool that is capable of supporting both. As a result, simulation modeling is dependent on the development of appropriate digital and mathematical tools. The same is true for previous modeling studies that required the use of digital tools (Brnic, and Greefrath, 2020; Greefrath, Hertleif and Siller, 2018; Greefrath and Siller, 2017; Schönbrodt, Wohak and Frank, 2022) and mathematical tools (Sinclair and Jackiw, 2010; Urgena and Lapinid, 2017) to achieve the desired results.

Learning activities on simulating deterministic behavior with Python were created for this study. The learning activities involved calculating the area under the curve and the area between the curves. The simulation of deterministic behavior in problems of area under curve and area between curves is crucial because it builds on knowledge of numerical methods for determining the finite integral of some functions that cannot be analyzed. The second gives an example of simulation modeling for deterministic problem situations (Albright and Fox, 2019; Giordano, Fox and Horton, 2013; Gordon and Guilfoos, 2017) although this type of modeling is commonly thought to be used for problems with probabilistic behavior. Python's selection as a mathematical tool should be due to all of the benefits of Python programs. That has evolved into a widely used programming language today. One of the anticipated outcomes of the activity was the improvement of students' competency in simulation for estimating the area between curves with Python. The second expectation is that students will experience enjoyment, value, interest, and self-efficacy in the activity.

Simulating deterministic Behavior with Python

This section describes the use of random number simulations to solve situations involving deterministic behavior in which certain outcomes are possible. In cases where there are no probabilities involved, the presented problem involves estimating the area between curves (Albright and Fox, 2019; Giordano, Fox and Horton, 2013; Gordon and Guilfoos, 2017). Consider the concept of the area under the curve enclosed by $y = f(x)$ as a continuous function, as shown in Figure 1, and consider the $0 \le f(x) \le M$ and $a \le x \le b$ boundaries.

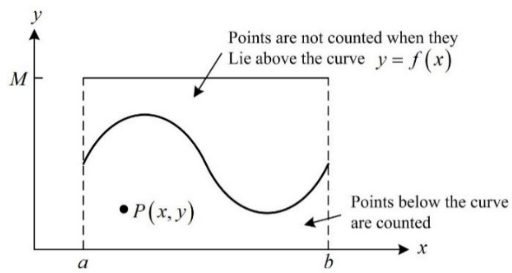

Figure 1. The rectangular region of high M and long $b - a$ encompasses the area from $y = f(x)$ to the $a \le x \le b$

Consider randomly selecting point $P(x, y)$ from the rectangular region. This is the result of generating random numbers x and y on $a \le x \le b$ and $0 \le y \le M$ and counting the point $P(x, y)$, which is located within the region whose area we wish to estimate.

A the area of the region to be estimated.
A_R the area of the rectangle.
P_C the number of random points on the area to be estimated.
P_T the total number of random points.
The relationship $\frac{A}{A_R} = \frac{P_c}{P_T}$ can be used to estimate the required area.

This process revealed that from the need to find the area of a region, rather than searching for tools to directly measure the area that must be known, we shifted the concept of the area problem to the idea that if we bound a rectangle, we know the exact area required to cover a region for the purpose of calculating its area value. Then it is comparable to throwing a dart over a rectangle containing the desired area. This means that the ratio of darts that hit the unknown area to the total number of darts thrown on the rectangle is equal to the ratio of the unknown area to the known rectangle's area. When converted to a mathematical process, this results in the creation of a random point on the rectangle containing the area to be evaluated. As a result, this is another study of the same problem that does not directly determine the value. This is what we call simulation modeling, and because it creates a simulation model to consider problem areas with deterministic behavior, we refer to it as simulating deterministic behavior.

The preceding concept can be used to create simulations to estimate the area between curves. To demonstrate the simulation process, we will present a simulation to estimate the enclosed space with $y = 4 - x^2, y = 3x, x = -4,$ and $x = 1.$ The procedure is as follows:

Analyzing a graph of area to be estimated

Because the Python code can approximate the area to be enclosed by $y = 4 - x^2$, $y = 3x$, $x = -4$, and $x = 1$, as shown in Figure 2, it is possible to consider generating a random point $P(x, y)$ on the $a \leq x \leq b$ and $0 \leq y \leq M$ rectangles with an area of 80 square units.

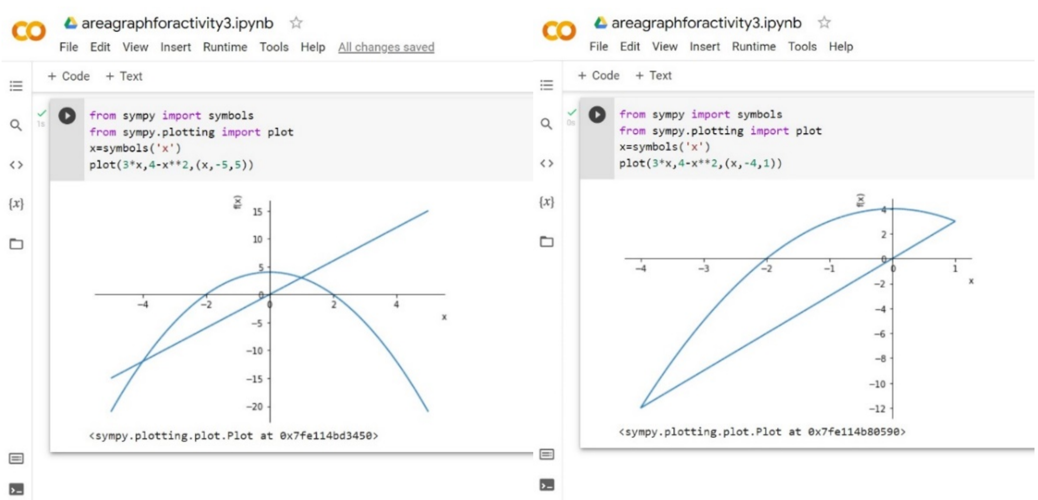

Figure 2. A graph of the area to be enclosed using the $y = 4 - x^2$, $y = 3x$, $x = -4$, and $x = 1$ equation on the left, and a graph of the $-4 \leq x \leq 1$ and $-12 \leq y \leq 4$ ranges on the right, which are boundaries to form a rectangle containing the estimated area.

Creating a simulation formula

When attempting to estimate the area that needs to be simulated, we can take into consideration the following formulas:

A represents the area of the area enclosed by $y = 4 - x^2$, $y = 3x$, $x = -4$, and $x = 1$.

A_R is the area of the rectangle $-4 \leq x \leq 1$ and $-12 \leq y \leq 4$.

P_C is the number of random points in the $y = 4 - x^2$, $y = 3x$, $x = -4$, and $x = 1$ region.

P_T represents the total number of random points on $-4 \leq x \leq 1$ and $-12 \leq y \leq 4$.

By considering all the variables, a formula $A = 80\frac{P_C}{P_T}$ can be developed that will be used to estimate the required area for simulation.

Writing the simulation algorithm

From analyzing the graph of the region to estimate the area and developing a formula that will be used in the simulation to estimate the area, an algorithm for commanding a computer can be written as shown in Table 1.

Table 1. The random number simulation algorithm used to solve the Area Between Curves problem.

A simulation algorithm for estimating the area between curves
Input The total number of random points n in the simulation on $-4 \leq x \leq 1$ and $-12 \leq y \leq 4$.
output The estimated value of the area enclosed by $y = 4 - x^2$, $y = 3x$, $x = -4$, and $x = 1$
START
Step 1 Set $counter = 0$
Step 2 For $i = 1,2,\ldots,n$ do Step 3-5
Step 3 Generate random coordinates x_i and y_i corresponding to $-4 \leq x \leq 1$ and $-12 \leq y \leq 4$.
Step 4 Calculate $f(x_i) = 4 - x_i^2$ and $g(x_i) = 3x_i$ for random point x_i
Step 5 If $g(x_i) \leq y \leq f(x_i)$ then $counter = counter + 1$ else $counter = counter$
Step 6 Calculate $Area = 80counter/n$
Step 7 OUTPUT rectangular region, region to estimate the area, all of the random points, and the estimated value of the area
STOP

Coding a Python for creating simulation to estimate an area

From the simulation algorithm written in Table 1, it is possible to write a Python program that will cause the computer to simulate the problem in order to find an estimate of the area whose code is displayed in Figure 3.

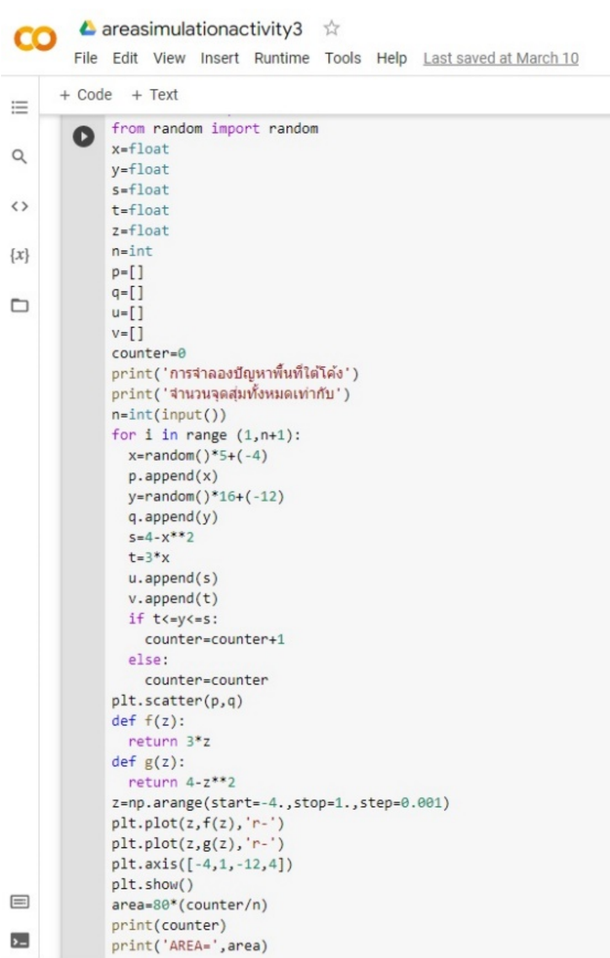

Figure 3. Python code for constructing a simulation to estimate the area of a region enclosed by $y = 4 - x^2$, $y = 3x$, $x = -4$, and $x = 1$.

Drawing a conclusion for simulation results

After generating the code that instructs Python to simulate the problem, the area between the curves enclosed by $y = 4 - x^2, y = 3x, x = -4$, and $x = 1$ is approximated. As depicted in Figure 4, a simulation was created in Python using 1000 random points.

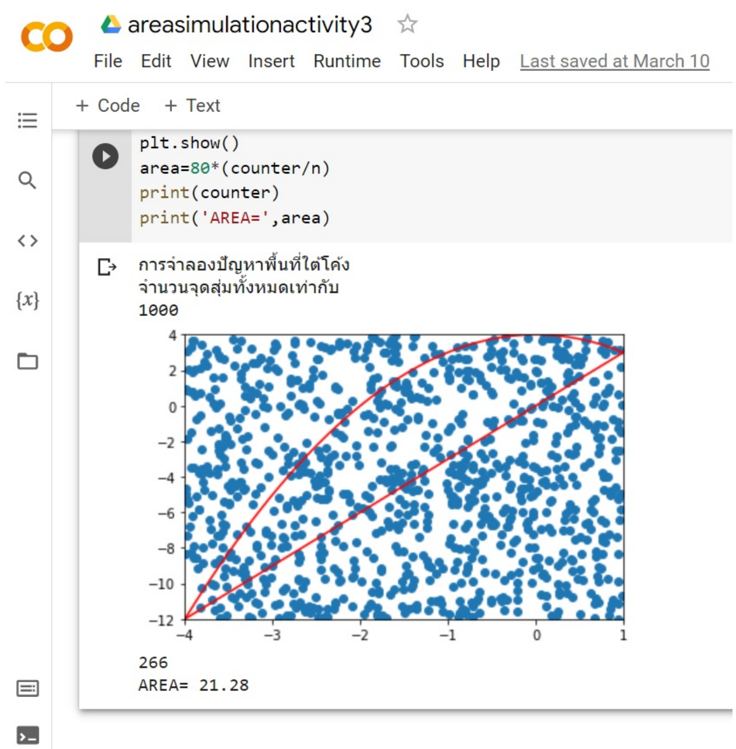

Figure 4. A random 1000-point numerical simulation for the area problem of a region bounded by $y = 4 - x^2, y = 3x, x = -4$, and $x = 1$.

The true value of the region bounded by $y = 4 - x^2$, $y = 3x$, $x = -4$, and $x = 1$ can be determined by using the definite integral, which is equal to $\frac{125}{6}$. Compared to the estimates derived from the simulations presented in Table 2, it is evident that a larger number of random points reduces the error. In order to create a simulation that provides a more precise estimate of the area, a large number of random points must be considered.

Table 2. The numerical solutions from simulations using increments of 100 from 100,000 random points to estimate the area of the enclosed region by $y = 4 - x^2, y = 3x, x = -4$, and $x = 1$.

Number of random points	Estimated Value	Absolute error
100	18.40000	2.43333
1000	21.84000	1.00666
10000	20.45600	0.37733
100000	20.80960	0.02373

By creating a simulation for estimating the area between curves with Python, as depicted in Figure 5, the simulation process competency can be reduced to five subcompetencies.

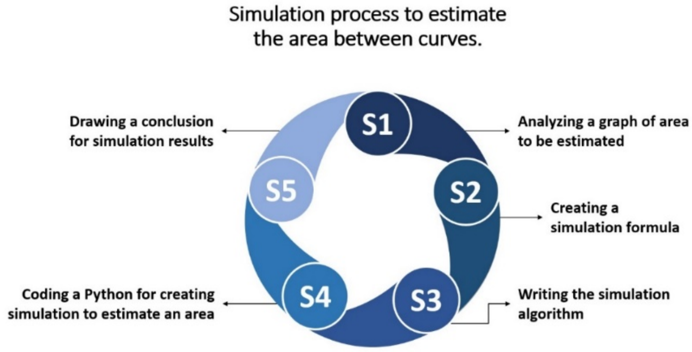

Figure 5. The five subcompetencies generated by the simulation process for estimating the area between curves using Python.

In this study, students will complete a python simulation activity to estimate the area between curves, with a list of all 6 activities summarized in Table 3.

Table 3. A list of activities and their characteristics for students to learn how to use Python to create a simulation for estimating the area between curves.

A list of activities	A characteristic of activities
Activity 1: Create a simulation to estimate the enclosed area by $y = x^2, y = x, x = 0$, and $x = 1$	Presentation of a simulation by a teacher as a demonstration
Activity 2: Create a simulation to estimate the enclosed area by $y = x^2 - 9, y = -x - 3, x = -3$, and $x = 2$	Presentation of a simulation by a teacher as a demonstration
Activity 3: Create a simulation to estimate the enclosed area by $y = 4 - x^2, y = 3x, x = -4$, and $x = 1$	The instructor guides the students through the process of creating a simulation.
Activity 4: A group of five or six students is tasked with determining the area to be estimated and developing a simulation to accomplish this.	Simulations are created through group activities.
Activity 5: Allow each student to create a simulation to estimate the enclosed region by equation $y = -x^3, y = -x, x = -1$, and $x = 1$	Individual tasks for the development of simulations
Activity 6: Create a simulation to estimate the enclosed area with $y = x^2, y = -x - 2, x = -1$, and $x = 2$	Activities for individual simulation competency testing

Scaffolding for Learning in the Age of Covid-19

This is due to the fact that estimating the area between curves through simulation is a new activity. It is essential to devise a strategy for scaffolding students in order for them to achieve their objectives. And because of the COVID-19 epidemic, it must manage online learning. Digital technologies must be integrated into the process of student scaffolding, which has been shown to improve learning management (Borba *et al.*, 2016; Clark-Wilson, Robutti and Thomas, 2020; Greefrath, 2011; Schukajlow, Kolter and Blum 2015; Stender and Kaiser, 2015; Tropper, Leiss and Hänze, 2015), which consists of the Facebook Group, Facebook

Messenger, Zoom, Google Form, and Google Collaboratory, utilizing these digital technologies to organize scaffolding for supporting students' learning in the following ways:

Teacher simulation-based activities

As students enter the classroom via the zoom channel, the instructor instructs them on how to construct a simulation to estimate the area between curves. It illustrates each step of the simulation and generates simulation-specific Python code via Google Collaboratory. Recorded videos of the activities are uploaded to Facebook groups for student review.

Simulation activities led by instructors.

After students have joined the class via Zoom, the instructor will direct everyone in the room to work together to develop a simulation that will estimate the area between the curves. The code for the simulation is written in Python and was generated using Google Collaboratory. Students are able to watch videos documenting the activities they participated in after they have been posted to Facebook groups.

Team-based creative simulation activities

The instructor divides 5-6 students into small groups in the Zoom Break Out Room as students enter the classroom via Zoom. Each student group created a problem by simulating the area between the curves and building a simulation to estimate the area between the curves. The Python code for the simulation is generated by the Google Collaboratory. Instructors enter the Zoom Break Out Room to provide group guidance. Students submit their work and video presentations via Google Forms upon completion, and instructors post videos on Facebook Groups to share their knowledge with all students.

Individual activities to create simulations

When students enter a Zoom classroom, the instructor directs them to individual sub-rooms in the Zoom Break Out Room, which are ordered chronologically. Every student creates a simulation to calculate the area between the curves. The Python code for the simulation is generated by the Google Collaboratory. Individual observation and assistance are provided by instructors, with a particular emphasis on assisting students with relatively slow cognitive development. Students submit their work via Google Form.

Individualized simulation competency testing

When students enter the classroom via the Zoom channel, the instructor provides them with a link to a Google Form simulation exam to estimate the area between curves.

Tools for Evaluating Learning Outcomes

The idea originated from a study on mathematical modeling competency assessment (Ferri, 2017; Lingefjärd and Holmquist, 2005) that investigated the effect of simulation activities on students' competency in simulation for estimating the area between curves using Python. According to the simulation creation procedure, competencies have been categorized into the five areas below:

S1: competency in analyzing a graph of area to be estimated
S2: competency in creating simulation formula
S3: competency in writing the simulation algorithm
S4: competency in coding a Python for creating simulation to estimate an area
S5: competency in drawing a conclusion for simulation results

The individual competency assessment is based on a consideration of the entire process using a scoring rubric that can reflect each student's individual achievement or competency level based on performance. Based on the quality of the students' grades, the rubric classifies the competency levels for each sub-competency into four categories: Excellent 4, Good 3, Satisfactory 2, and Needs Improvement 1 Score.

For the emotional promotion activity expectations, four evaluations were conducted: enjoyment, value, interest, and self-efficacy. The Likert scale questionnaire was utilized to assess the impact of the simulation activity for estimating the area between curves with Python on the students. This includes updated questions from a research study on the subject (Krawitz and Schukajlow, 2018). The following is a summary of the questions:

Q1: I would enjoy solving simulation problems to estimate the area between curves.

Q2: I think it is important to be able to solve simulation problems to estimate the area between curves.

Q3: It would be interesting to investigate simulation problems in order to estimate the area between curves.

Q4: I am confident in my ability to solve simulation problems to estimate the area between curves.

Research Methodology

This research aims to promote secondary school students' competency in fundamental simulation modeling. This will be the initial performance that will be further developed at a higher level. The research methodology section consists of participants involved in the study, research tools, data analysis approach, and research process, which can be summarized as follows:

Participants involved in the study

The participants in the study were secondary school students who took part in the Science Classrooms in the University-Affiliated School Project. There was a total of 28 students in Grade 12 who participated in the project, and their ages ranged from 17 to 18 years old.

Research tools

The research tool consists of learning activities on simulation for estimating the area between curves with Python, the Python program as a mathematical tool, learning tools such as Facebook, Zoom, and Google, and learning devices such as smart phones, tablets, laptops, and personal computers based on student needs. In terms of data collection, it consists of an assessment of the students' simulation competences for estimating the area between curves with Python. This evaluation takes the form of a subjective test given at the end of the activity. Finally, we used the Likert Scale questionnaire to assess the effects of the activities on enjoyment, value, interest, and self-efficacy.

Data analysis approach

The rubric results comprise the analytical approach. The data on the frequency of learners in each competency level within each sub competency is presented. The overall level of competence in each sub competency is summarized using the mean scores of students. The mean scores and standard deviations of the Likert Scale were presented to identify students' levels of enjoyment, value, interest, and self-efficacy as a result of activities.

Research process

The process involves developing and preparing research tools, considering the preparation of the target audience, performing simulation activities to estimate the area between the curves with Python and the target audience according to the planned scaffolding strategy, and analyzing the results to draw conclusions.

Results and Discussion

To demonstrate how this developed activity can improve students' competency in simulation for estimating the area between curves with Python, along with activities that can increase students' enjoyment, value, interest, and self-efficacy, an evaluation case of one student's work will be presented to visualize the work representing the level of competency of each sub-competence that is a key component of the simulation for estimating the area between curves with Python. The results of the rubric assessment of competency in simulation for estimating the area between curves with Python will be presented, followed by the results of the students' enjoyment, value, interest, and self-efficacy. They can be presented in the following order based on the foregoing:

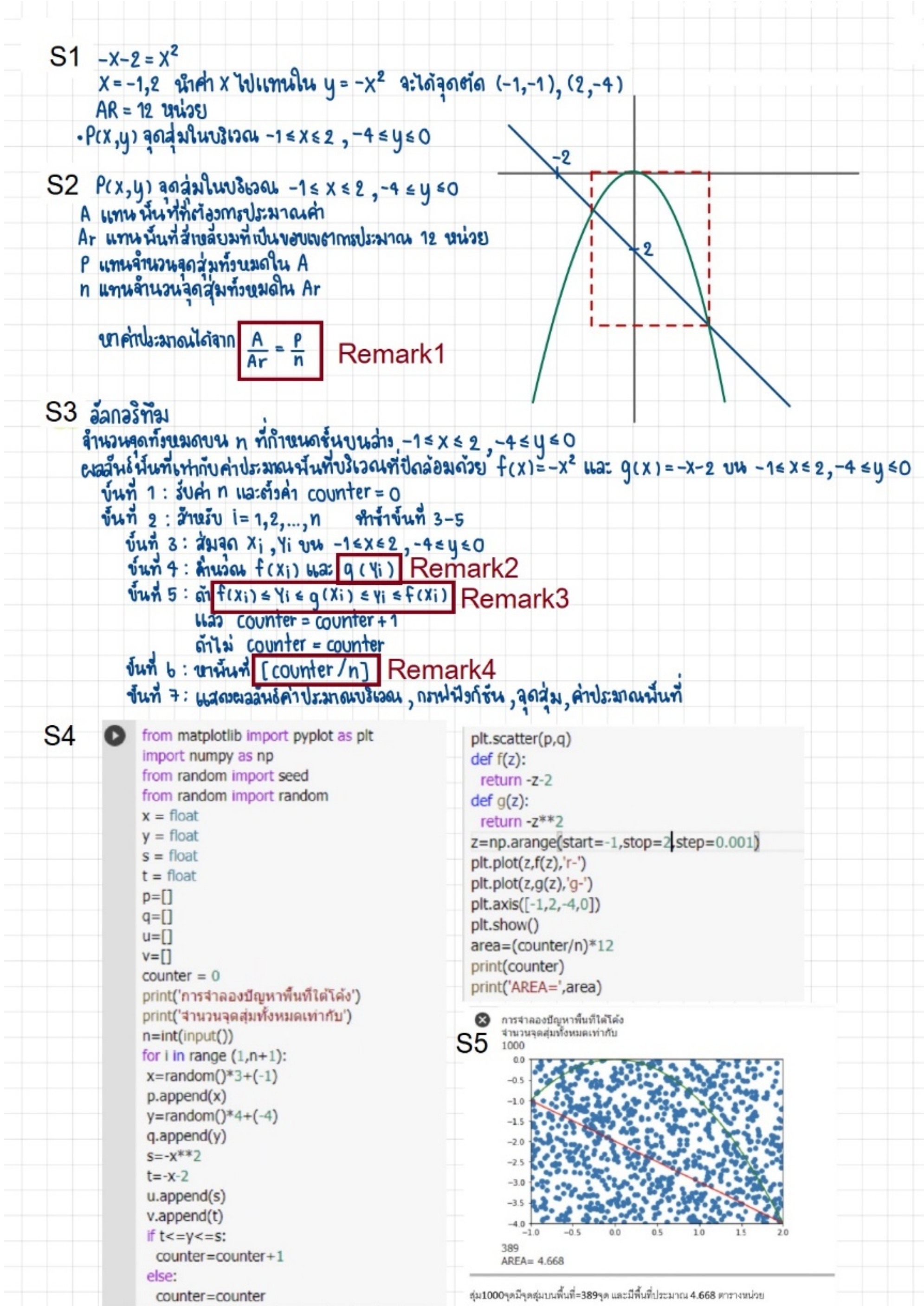

Figure 6. The pictorial representations for competency levels of students for each sub-competency of simulation for estimating the area between curves with Python

Consider Figure 6, which cites the work of one student. This concretely reflects the results of the simulation competency assessment for estimating the area between curves with Python. Labels S1, S2, S3, S4, and S5 were added to indicate the work position corresponding to each sub-competency that is an important component of the Python simulation to estimate the area between curves, and Remarks 1, 2, 3, and 4 were added for further reference. The outcomes of each sub-competency assessment are as follows:

S1: competency in analyzing a graph of area to be estimated

Figure 6 demonstrates that students can analyze the graph of the desired area to estimate the area completely correctly. Python is used to generate a graph of the area to be estimated between the $y = -x^2$

and $y = -x - 2$. A rectangular boundary is drawn around the area between the curves. The results are correct and can be used in the next step. As a result, S1's sub-competence evaluation result is excellent, with 4 points.

S2: competency in creating simulation formula

S2 sub-competency assessment results revealed that students were able to correctly construct a simulation formula of the desired area to estimate the area from the results obtained in the previous step. However, taking into account Figure 6 Remark 1, the formula $\frac{A}{A_R} = \frac{P}{n}$ remains a general formula. It is considered incomplete because it cannot be used immediately in the next step. In this case, the formula should be changed to $A = 12\frac{P}{n}$, yielding a score of 3 Good.

S3: competency in writing the simulation algorithm

When considering the sub-competence in this area, this is the subject that still needs further development. Students understand the steps to be programmed to create a simulation but still lack the ability to write algorithms. The difficulty of writing an algorithm comes from the fact that each step is logically put together. The language used to write algorithms is a language with logical and mathematical meanings involved, which is difficult for students. Another is that students are unaware of the importance of writing algorithms. Students do not know that algorithms are very important in the development of other programs. Therefore, it is not important to ask how the program is created correctly or what the algorithm will be. In S3, at Remark 2 position, $g(y_i)$ should be adjusted to $g(x_i)$, at Remark 3 position $f(x_i) \leq y_i \leq g(x_i) \leq y_i \leq f(x_i)$ is invalid, it must be solved to $g(x_i) \leq y_i \leq f(x_i)$ and finally Remark 4 Enter the formula for finding the wrong area should be revised from $\frac{counter}{n}$ to $12\frac{counter}{n}$, the assessment result is in Needs Improvement level, score is equal to 2.

S4: competency in coding a Python for creating simulation to estimate an area

According to Figure 6, the simulation python code for estimating the area between the curves of the students was at an excellent level, representing 4 points, despite the fact that the algorithm writing in the previous step was incomplete. As previously stated, this demonstrates that students do not understand the significance of writing the algorithm.

S5: competency in drawing a conclusion for simulation results

As for the S5 sub competency, there is one thing that needs to be said about the number of simulations to be considered. A simulation for estimating the area between curves will give more accurate values if using a large n. In the simulation, the student selected the number $n = 1000$. The simulation result is shown in Figure 6, which shows a graph of the area to be estimated along with the points. randomly generated in the simulation. A total of 389 random points are shown in the area to be estimated. The area value is presented as 4.668 compared to the actual value obtained by the integral $\int_{-1}^{2}(-x^2) - (-x - 2)\,dx = 4.5$ is still a little off, possibly due to the increased number of simulations. The evaluation results are therefore at the Good level with a score of 3.

It is possible to deduce, based on the findings of the evaluation of competency in simulation for estimating the area between curves with Python, that the number of students who achieve each sub-competency in simulation for estimating the area between curves with Python corresponds to the number of students who achieve the competency level depicted in Figure 7.

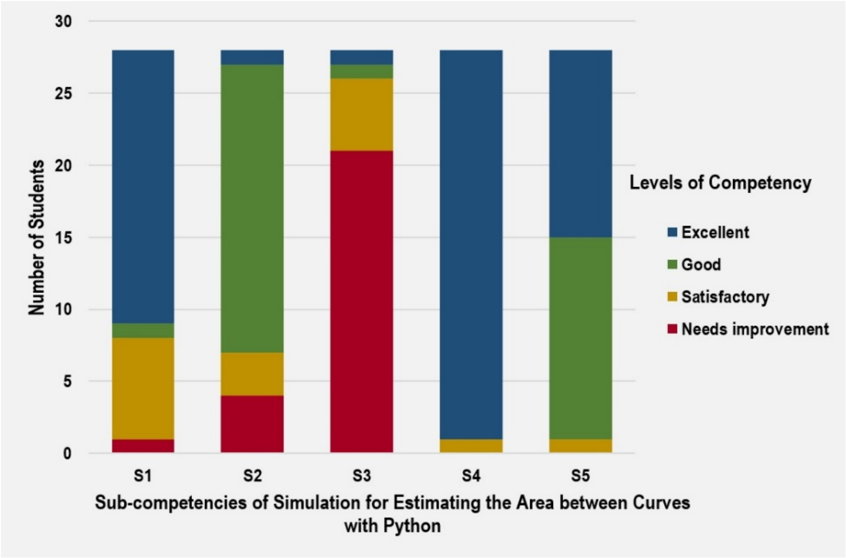

Figure 7. The number of students at each competency level for each sub-competency of simulation for estimating the area between curves with Python

The results of the sub-competences can be summarized based on the evaluation results shown in Figure 7: S1 is the competency in analyzing a graph of the area to be estimated. It was discovered that the majority of the students performed at an excellent level. Aside from that, only a small number of students were at the level of needs improvement, implying that teachers needed to consider how to improve further.

S2 is competency in creating simulation formulas for the area to be estimated. The majority of students were graded as Good, with only a small number receiving an Excellent rating, indicating that there were minor errors in the process that teachers could easily add to their knowledge and correct. There are a number of students ranging from satisfactory to needs improvement for whom instructors should reconsider this sub-competence development approach.

S3 is competency in writing a simulation algorithm to estimate the area between curves. The majority of the students were found to be in need of improvement. Only a small number of students performed the activities at a satisfactory or higher level, despite the fact that they were able to complete the process and achieve good simulation results. However, developing this sub-competency is absolutely necessary. This is due to the fact that it is a critical factor that influences the development of other advanced competencies.

S4 is competency in coding a Python for creating simulation to estimate an area. It was discovered that the majority of the students performed at an excellent level, with only a small number performing at a satisfactory level, which was a minor error that teachers could easily correct. Most students agree that writing Python is enjoyable. As a result, there is a lot of motivation behind this activity. Students are consulting with one another. A meeting with the teacher is required. The activities promote knowledge sharing. As a result, the performance test results in this sub-competency section are favorable.

S5 is competency in drawing conclusions from simulation results based on estimating the area between curves. It was discovered that all students had competency levels of satisfactory or higher, which, based on the results of previous activities, affected this level of sub-competency.

The results of the assessment of the students' competency level in each sub-competence can be assessed as a score using the rubric criteria to find the mean to represent the overall picture of the student's level in each of the sub-competences in simulation for estimating the area between curves with Python, as shown in Figure 8.

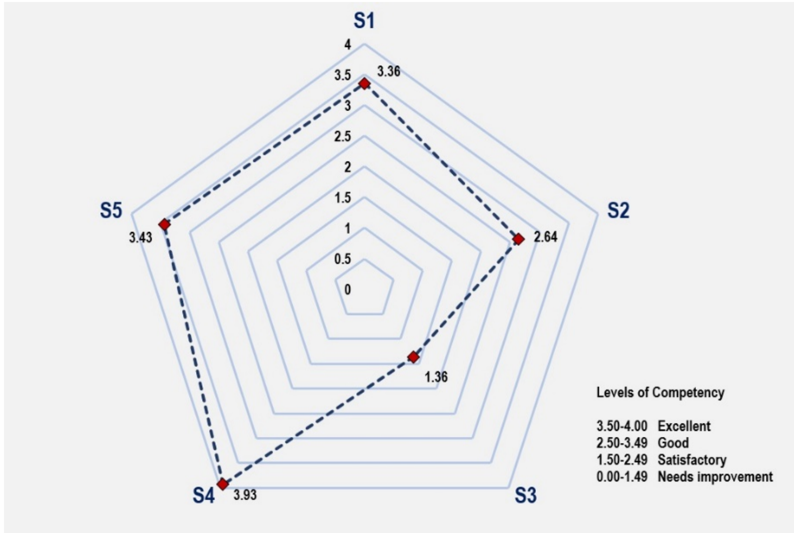

Figure 8. The mean score for competency levels of students for each sub-competency of simulation for estimating the area between curves with Python

Figure 8 depicts students' simulation competency for estimating the area between curves with Python. Overall, the sub-competency S4 had a mean score of 3.93, higher than 3.5, indicating excellent competency in coding Python for creating simulations to estimate an area, whereas the sub-competency S1, S2, and S5 had mean scores of 3.36, 2.64, and 3.43, respectively, between 2.5 and 3.5. These results indicate that the competency in analyzing a graph of an area to be estimated, creating a simulation formula to estimate the area, and drawing conclusions from simulation results is Good. Finally, the sub-competency S3 averaged 1.36, indicating that competence in writing the simulation algorithm to estimate the area between curves is lacking and needs to be developed further.

The results demonstrate the success of using simulation enhancement activities to promote simulation competency for students estimating the area between curves with Python. Consequently, for starters, students in Science Classrooms in University-Affiliated School Project perform academically better than average students. The second result is the use of digital technology to assist in the planning of a good scaffolding strategy, which is consistent with the findings of several studies that describe the effectiveness of digital technology in promoting good learning outcomes in mathematics (Borba *et al.*, 2016; Clark-Wilson, Robutti and Thomas, 2020; Greefrath, 2011; Schukajlow, Kolter and Blum 2015; Stender and Kaiser, 2015; Tropper, Leiss and Hänze, 2015). Finally, using Python in simulation is an important tool provided by the Google Collaboratory. It is more conducive to student learning because it is more convenient and efficient. It is similar to a mathematical tool program whose researchers have previously reached conclusions about the promotion of student learning (Sinclair and Jackiw, 2010; Urgena and Lapinid, 2017).

The results of the evaluation of the simulation for estimating the area between curves in Python based on the learners' enjoyment, value, interest, and self-efficacy are presented in Table 4.

Table 4. The assessment results of learning activities on the simulation for estimating the area between curves with Python on learners' enjoyment, value, interest, and self-efficacy.

items	Evaluation lists	Mean	Standard deviation	Results
Q1	I would enjoy solving simulation problems to estimate the area between curves.	4.39	0.89	agree
Q2	I think it is important to be able to solve simulation problems to estimate the area between curves.	4.30	0.76	agree
Q3	It would be interesting to investigate simulation problems in order to estimate the area between curves.	4.13	1.06	agree
Q4	I am confident in my ability to solve simulation problems to estimate the area between curves.	3.91	1.12	agree

According to Table 4, the students agreed that the activity increased their enjoyment, value, interest, and self-efficacy in creating a simulation in Python to estimate the area between curves. This indicates that the simulation activities for estimating the area between curves with Python have a positive effect on the four desired emotional characteristics of the student.

Python simulation for estimating the area between curves Activities that emphasize learners' group processes and strategies for providing assistance and empowering students are utilized. Multiple studies have concluded that this type of activity has a positive impact on students' attitudes (Davadas and Lay, 2018; Schukajlow *et al.*, 2012).

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