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Research Article

Decision-making method for multi-ship collision avoidance based on improved extensive game model

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Abstract

In the case of multi-ship encounter situations, most collision accidents are caused by not having timely and coordinated collision avoidance behaviors. Therefore, it is necessary to introduce an automatic collision avoidance method in the process of collision avoidance to quickly generate multi-ship collision avoidance decisions based on encounter situations. Multi-ship collision avoidance is a dynamic game used among officers under situations of multi-ship encounters. Considering the defects of the classic extensive game method in ship collision avoidance decision-making, the improved extensive game method is proposed, based on the velocity obstacle method. The feasible region of velocity is taken as the optional action range of the game to narrow down the action set. The simulation results show that the proposed method can effectively transform the multi-ship collision avoidance game problem into a 2-ship collision avoidance game problem, reduce the optional action range of a collision avoidance action set, and improve decision-making speed, while ensuring the safety of multi-ship navigation.

List of symbols

N	Player set
O	Own ship
TS_{\max}	Ship that has greatest collision risk with own ship
S	Action set
s	Action
t	Time interval for decision-making
T	Moment
V	Speed
ψ	Course
Δx	Change in x-coordinate
Δy	Change in y-coordinate
q	Round number of game

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(x_0, y_0)	Initial position
(x_p, y_p)	Position reached after t seconds of sailing on original course
Δx_p	Change in x-coordinate within t seconds of sailing on original course
Δy_p	Change in y-coordinate within t seconds of sailing on original course
(x_q, y_q)	Position reached after t seconds of sailing on decided course
Δx_q	Change in x-coordinate within t seconds of sailing on decided course
Δy_q	Change in y-coordinate within t seconds of sailing on decided course
$payoff_q$	Payoff of ship
(a_1, b_1)	Position of own ship at time $T-1$
(a_2, b_2)	Position of own ship at time T
TS	Target ship
I	Number of target ship
(a_3, b_3)	Position of ship at time T
V_o	The velocity of own ship
V_{TS}	The velocity of target ship
V_{OT}	The velocity of own ship relative to target ship
m	Total number of layers in game tree
i	Number of layers in game tree
j	Node number of game tree
k	Number of actions in game tree
f_{ij}^N	State space of layer i , node j
s_{ik}^N	Action k that ship can take when making decisions of collision avoidance at nodes of layer i
u_{mj}^N	Payoffs of ships at layer m , node j
l	Node number of ship with greatest payoff
D_x	Value of ship domain on x-axis of inertial coordinate system
D_y	Value of ship domain on y-axis of inertial coordinate system
(x_l, y_l)	Center of gravity coordinates of ship.
K_p	Proportion parameter of incremental PID control method
K_i	Integral parameter of incremental PID control method
K_d	Differential parameter of incremental PID control method
ψ_{goal}	Target course

1. Introduction

In recent years, the world has witnessed a dramatic increase in maritime trade, traffic, and ship tonnage, which causes much more complex maritime traffic environments in some waters. Especially during multi-ship encounters, complicated situations will result in ship collision accidents. As a consequence, it is urgent to develop scientific research on the approaches to collision avoidance, which can provide decision-making support for multi-ship collision avoidance to coordinate behaviors among ships, and substantially reduce the occurrence of collision accidents.

In multiple ship encounters, the process of collision avoidance is highly linked with the result of game and its interaction among ships. Based on its characteristics, it is of great significance to study the collision avoidance of ships using the framework of game theory. When the classic

extensive game model is applied to multi-ship collision avoidance, the collision risk is measured by the risk model to determine the game order. The action set is screened based on experience, and a game tree is created based on all ships in the encounter situation. As the number of ships increases, the number of game tree layers and nodes increases exponentially, resulting in a significant decrease in the efficiency of collision avoidance decision-making. In this context, the paper presents a multi-ship collision avoidance algorithm based on improved extensive game to provide theoretical and technical support for researches on multi-ship collision avoidance.

On the basis of in-depth study of ship collision avoidance mechanisms, an improved extensive game model of ship collision avoidance is presented. The method of screening players and action sets in multi-ship encounter situations is improved based on velocity obstacle. An algorithm to find the most dangerous target ship is designed. Based on the velocity feasible region of the ship avoiding the most dangerous target ship and its surrounding ships, the optional action range of the collision avoidance game model is determined, which can avoid the risk of collision with the third ship after the own ship turns to avoid collision. Consequently, the multi-ship collision avoidance game is transformed into a 2-ship collision avoidance game on the premise of ensuring navigation safety. The offset of ship is taken as the payoff function, and the corresponding payoffs are calculated according to the action sets of the 2 ships in the game. A game tree of ship collision avoidance is created. The subgame perfect Nash equilibrium is solved by backward induction, which makes 2 ships choose the most beneficial decisions for themselves, namely, the decisions with the largest payoffs, to form a collision avoidance strategy profile.

In this paper, ship collision avoidance maneuvering has been simulated on the basis of the Ship Maneuvering Mathematical Model Group (MMG) model, and the ship's course is controlled by an incremental PID controller. Encounter situations are derived by maneuvering simulation combined with a multi-ship collision avoidance algorithm under the framework of dynamic game. The presented method is compared with the classic extensive game algorithm from 2 key aspects: action set screening and CPU time consumption. The performance of the algorithm in the whole collision avoidance process is analyzed and its effectiveness is verified.

The rest of the paper is arranged as follows: Section 2 provides a literature review of related publications; Section 3 presents the game model of ship collision avoidance; in Section 4, details of the action set screening algorithm based on velocity obstacle are elaborated; in Section 5, the solution method of optimal action profile is carried out; in Section 6, the process of improved extensive game method is presented; in Section 7, the multi-ship collision avoidance simulation experiment is carried out; Section 8 concludes the main findings and provides future research directions.

2. Literature review

Polish academic Lisowski has made important contributions to the application of game theory in the field of ship collision avoidance. In 2004, Lisowski combined a multi-step matrix game model with optimal online control to determine the safety trajectory of ships (Lisowski, 2004). The International Regulations for Preventing Collisions at Sea (COLREGS) were taken into account in the method. It was pointed out that the number of strategies that can be adopted by ships will have an essential influence on the formation of the safe optimal trajectory and the final deviation between the game trajectory and the reference trajectory. In 2007, Lisowski introduced the multi-stage position game model on the basis of the multi-step matrix game model and simulated a ship's meeting with many target ships in environments of good visibility and limited visibility. The results showed that the model could still determine the safe game trajectory of ships (Lisowski, 2007). In 2008, Lisowski (2008a) applied the multi-step matrix game model to the ship navigation problem in crowded waters. He proposed a game control method named RISKTRAJ, based on the previous model, to determine the safe game trajectory of a ship in the situation of a multi-ship encounter. In addition, the sensitivity of ship game control to inaccurate information

from Automatic Radar Plotting Aid (ARPA) and parameter changes in the control process was analyzed. In the same year, Lisowski proposed a game control method named POSTRAJ, based on the multi-stage position game model (Lisowski, 2008b). In 2010, Lisowski analyzed the sensitivity of the RISKTRAJ and POSTRAJ algorithms in actual sea navigation based on data from ARPA radar (Lisowski, 2010). In 2012, on the basis of the multi-stage position game control algorithm and multi-step matrix game control algorithm, a cooperative game model and non-cooperative game model were introduced to simulate the collision avoidance of the 4 algorithms in the situation of a multi-ship encounter (Lisowski, 2012). COLREGS and the advance of ship maneuvering were taken into account in the algorithm. The positional game control algorithm determined the safe game trajectory for the own ship to avoid all other ships in a multi-ship encounter situation. The matrix game control algorithm determined the safe game trajectory for the own ship to avoid the most dangerous ship. In 2013, Lisowski analyzed the sensitivity characteristics of a multi-stage non-cooperative positional game algorithm, a multi-stage cooperative positional game algorithm, and a kinematic optimization control algorithm through examples of navigation with limited visibility at sea (Lisowski, 2013).

Szlupczynski is another representative scholar who uses game theory to solve the problems of ship collision avoidance. In 2011, Szlupczynski combined some assumptions of game theory with evolutionary programming and presented a method to solve the safe trajectory sets of all ships in a multi-ship encounter situation, named Evolutionary Sets of Safe Ship Trajectories (ESoSST) (Szlupczynski, 2011). It could solve the problem of multi-ship collision avoidance in open and restricted waters. The method not only provided the optimal collision avoidance strategy for the own ship, but also planned the optimal collision avoidance trajectory in a multi-ship encounter situation. Considering COLREGS constraints, the total route loss of trajectory calculation was minimized, and the real-time performance of the algorithm was good. Then, considering the provisions of Traffic Separation Schemes (TSS) in Rule 10 of COLREGS, the ESoSST method was improved (Szlupczynski, 2012; Szlupczynski, 2013). First, trajectory sets that followed regulations within TSS were predefined. Then, a fitness function was designed to evaluate the trajectory sets, which included penalties for TSS violations. Finally, special evolutionary operators were used to eliminate the violation of rules by adjusting the trajectory of the ship. In 2015, Szlupczynski continued to expand the ESoSST method based on the provisions of ships sailing on waters with restricted visibility in Rule 19 of COLREGS (Szlupczynski, 2015). The ESoSST method is a centralized method to solve collision avoidance trajectory, which avoids the occurrence of secondary collision risk caused by uncoordinated actions in multi-ship collision avoidance. It is suitable for the generation of a collision avoidance scheme of Vessel Traffic Service (VTS) system.

Chinese scholars who research the application of game theory in the field of ship collision avoidance are relatively few. Zhang and Shi (2007) transformed the problem of ship collision avoidance in close-quarter situations into a differential game and studied the collision avoidance strategy of the own ship when the target ship takes actions unfavorable to collision avoidance on the premise that the own ship and the target ship keep a uniform motion. Wang (2014) and Ouyang et al. (2020) regarded ship collision avoidance as a complete information dynamic game between ships and established a decision-making model of ship collision avoidance based on extensive game. Wang took the velocity and gross tonnage of ships as the basis to determine the priority of collision avoidance. Ouyang used Distance to Closest Point of Approach (DCPA), Time to Closest Point of Approach (TCPA), and other parameters to calculate the collision risk between 2 ships, based on which the priority of collision avoidance was determined. In the former model, the payoff function was the offset of ship, while in the latter, the payoff function was constructed by DCPA, TCPA, offset, and handling deviation of the ship. The former studied 2-ship collision avoidance, while the latter converted multi-ship collision avoidance into 2-ship collision avoidance in stages; on the basis of the former, decision-making for ship collision avoidance in multi-ship encounter situations was realized.

3. Game model of ship collision avoidance

In the process of multi-ship collision avoidance, every officer observes and analyzes the navigation behavior of other ships, then makes the most beneficial decision of collision avoidance while ensuring their own safety. Multi-ship collision avoidance is a dynamic game among officers under the situation of multi-ship encounter. When an officer maneuvers the ship, the order is agreed, and collision avoidance actions are taken under the constraint of COLREGS. Players who take action later know the previous game and the decisions of the former players. In this paper, the extensive game model is used to describe ship collision avoidance. In multi-ship encounter situations, the principle of velocity obstacle is introduced to determine the ships that need to be avoided and the actions needed to be taken by each ship, and an improved extensive game method of multi-ship collision avoidance is proposed.

Based on the decision-making process of collision avoidance for each ship in multi-ship encounter situations, the collision avoidance problem is defined by the extensive game model.

Player set: A player is a participant in the game. In the game model of ship collision avoidance, player set is $N = \{O, O_{\max}\}$, where O represents the own ship and O_{\max} represents the ship that has the greatest collision risk, namely, the main target ship. In this paper, the main target ship is determined by the principle of velocity obstacle (Section 4).

Action set: In a dynamic game between ships, the set of all actions that a ship can choose when she makes a decision is called the action set of the ship, represented as $S = \{s_1, s_2, \dots, s_n\}$. The velocity obstacle method is used to screen the action set, then an appropriate interval is selected to further divide the optional range (Section 4). There are 3 main ways for ships to avoid collisions at sea: turning, decelerating, and stopping. According to COLREGS, turning alone may be the most effective action to avoid a close-quarters situation, so this paper only considers turning, instead of decelerating, stopping, or other methods to avoid collision.

The order of play: The order of play is the order in which decisions are made. At any stage of the collision avoidance game, ships make the choice of action in a certain order, and the those that take action later know the previous game and the decisions of the former players. In this paper, it is prescribed that the ship will make the decision of collision avoidance first if she turns and does not pose a risk of secondary collision with other ships in a multi-ship encounter situation. The latter ship makes the decision based on the movement of the former ship, and 2 ships make decisions alternately. The order of play is $O \rightarrow O_{\max} \rightarrow O \rightarrow \dots \rightarrow O_{\max}$.

Information set: An extensive form of game is also called a game tree. In a game tree, the information set is represented by a series of decision nodes, describing the stage of the game and the decisions made by the former ship. In the proposed algorithm, each node of the game tree contains the following information: the ship's speed, course, position, payoff, and the other ships' action sets. In each round of the game, every ship involved in the game knows the actions of collision avoidance taken by the other ships in real time.

Payoff function: Payoff is a function that measures the preference of collision avoidance actions taken by the ship (the player). Collision avoidance decisions are different based on different payoff functions. In the situation of collision danger, the officers generally choose a turning angle to avoid collision which is as small as possible, based on safe avoidance. In this paper, velocity obstacle is used to screen the feasible set of collision avoidance. In the feasible set, all actions of a ship can make it pass another ship safely. Based on the decision-making tendency of officers to avoid collision, offset of ship serves as the payoff in this paper, and the payoff function is constructed. The smaller the offset of ship is, the smaller the cost to avoid collision is, and the greater the payoff of ship is.

In time interval t , the payoff function of the ship is as follows:

Change in the x-coordinate Δx :

$$\Delta x = \begin{cases} Vt \cos(\psi) & 0^\circ \leq \psi \leq 90^\circ \\ -Vt \sin(\psi - 90^\circ) & 90^\circ < \psi \leq 180^\circ \\ -Vt \cos(\psi - 180^\circ) & 180^\circ < \psi \leq 270^\circ \\ Vt \sin(\psi - 270^\circ) & 270^\circ < \psi < 360^\circ \end{cases} \quad (1)$$

Change in the y-coordinate Δy :

$$\Delta y = \begin{cases} Vt \sin(\psi) & 0^\circ \leq \psi \leq 90^\circ \\ Vt \cos(\psi - 90^\circ) & 90^\circ < \psi \leq 180^\circ \\ -Vt \sin(\psi - 180^\circ) & 180^\circ < \psi \leq 270^\circ \\ -Vt \cos(\psi - 270^\circ) & 270^\circ < \psi < 360^\circ \end{cases} \quad (2)$$

where V is the speed of the ship. It is assumed that the ship travels at a constant speed. ψ is the course of the ship.

Because the payoff function of the ship involves the identity transformation of trigonometric function and the mathematical property will lead to the confusion of calculation of course angle in programming, it is necessary to convert the course angle to $0 \sim 2\pi$. The conversion formula is shown as follows:

$$\psi = \begin{cases} \psi & 0^\circ \leq \psi < 360^\circ \\ \psi - 360^\circ & \psi \geq 360^\circ \\ \psi + 360^\circ & \psi < 0^\circ \end{cases} \quad (3)$$

The ship does not take the decision of collision avoidance game in round q , and arrives at the position (x_p, y_p) after staying on course for t seconds:

$$\begin{cases} x_p = x_0 + \Delta x_p \\ y_p = y_0 + \Delta y_p \end{cases} \quad (4)$$

where (x_0, y_0) is the initial position of the ship. Δx_p and Δy_p represent the changes in horizontal and vertical coordinates, respectively, of the ship in time t .

The ship takes the decision of collision avoidance game in round q , and arrives at the position (x_q, y_q) after t seconds sailing on the decided course:

$$\begin{cases} x_q = x_0 + \Delta x_q \\ y_q = y_0 + \Delta y_q \end{cases} \quad (5)$$

where Δx_q and Δy_q represent the changes in horizontal and vertical coordinates, respectively, within t seconds of the ship sailing on the course for the decision in round q .

The payoff function of the ship in round q is calculated in formula (6):

$$payoff_q = -\sqrt{(x_q - x_p)^2 + (y_q - y_p)^2} \quad (6)$$

4. Action set screening algorithm based on velocity obstacle

The collision avoidance actions of all ships in the situation are calculated based on a game tree when multi-ship collision avoidance is studied by the classic extensive game method. As the number of ships increases, the number of nodes in the game tree increases exponentially. As a result, the computational resources consumed to solve the collision avoidance behavior also increase. In this paper, an improved extensive game method is proposed, which is based on the velocity obstacle method to determine the dangerous ship and the action set that each ship needs to take. The multi-ship collision avoidance game is transformed into a 2-ship collision avoidance game. Using the algebraic judgment advantage of the velocity obstacle method, the screening speed of the dangerous ship and action set can be improved, and the possibility of collision with the third ship can be avoided.

The basic idea of the action set screening algorithm based on the velocity obstacle method is as follows: firstly, determine the velocity obstacle area. Then, screen optional action range according to the multi-ship motion situation. Finally, apply equal interval division to the optional action range, select the action, and determine the action set.

4.1 Ship domain

Multi-ship encounter situations often occur in coastal waters, which are complex and changeable. Ships frequently change course to avoid collision. In this paper, the action of ship collision avoidance is based on the action set. The action set is directly related to the velocity obstacle area. The ship domain directly affects the range of the velocity obstacle area. As a consequence, the ship domain affects the determination of the ship collision avoidance action. Ship domain is the safe distance that a ship should keep with other ships. The larger the ship domain, the farther the distance between ships in the encounter situation, and the safer the sailing. To ensure the same safety of the ship's bow, stern, and both sides, a circular ship domain centered on the ship's barycenter is constructed in this paper.

The ship domain is modeled as a circle with a radius of $radius$ m. The formula for the ship domain is as follows:

$$\begin{cases} D_x = x_I + radius \cdot \cos \theta \\ D_y = y_I + radius \cdot \sin \theta \end{cases} \quad (7)$$

where D_x is the value of the ship domain on the x-axis of the inertial coordinate system, D_y is the value of the ship domain on the y-axis of the inertial coordinate system, and $\theta \in (0, 2\pi)$. (x_I, y_I) are the center of gravity coordinates of ship.

4.2 Dangerous ship detection

Each ship takes turns as the own ship to conduct the collision detection on other ships in the encounter situation. The target ship is the other ship in the multi-ship encounter situation relative to the own ship. Assume that the own ship is located at $Loc_1(a_1, b_1)$ at time $T-1$, and is located at $Loc_2(a_2, b_2)$ at time T . Target ship TS_i is located at $Loc_3(a_3, b_3)$ at time T . If the target ship meets the 3 conditions at the same time, she will be considered to be dangerous and to pose a risk of collision with the own ship:

1. The distance between the target ship and the own ship is less than the threshold, i.e.,

$$\sqrt{(a_2 - a_3)^2 + (b_2 - b_3)^2} \leq 3nm \quad (8)$$

2. The target ship is ahead of the own ship

To determine whether the target ship is in front of the own ship, first of all, make a straight line fb perpendicular to $Loc_1 Loc_2$ through point Loc_1 :

$$fb = -\frac{a_2 - a_1}{b_2 - b_1}(x - a_1) + b_1 \quad (9)$$

Then, determine whether points Loc_2 and Loc_3 are on the same side of the line. The coordinates of point Loc_2 and point Loc_3 can be substituted into fb , respectively, and by multiplying expressions, the discriminant is as follows:

$$fc = \left[\frac{a_2 - a_1}{b_2 - b_1}(a_2 - a_1) + b_2 - b_1 \right] \left[\frac{a_2 - a_1}{b_2 - b_1}(a_3 - a_1) + b_3 - b_1 \right] \quad (10)$$

If $fc > 0$, point Loc_2 and point Loc_3 are on the same side of fb , the target ship is ahead of the own ship; if $fc < 0$, point Loc_2 and point Loc_3 are on different sides of fb , the target ship is astern of the own ship.

3. The velocity direction of own ship O relative to the target ship TS_i falls into the velocity obstacle area $A_i OB_i$. The judgment method is as follows: make the extension line lv_i of the relative velocity V_{OT_i} direction. Judge whether lv_i intersects with the ship domain of the target ship TS_i .

The judgment method of the most dangerous ship is as follows: if there is more than one other ship which is in danger of collision with the own ship, a ship which is on the starboard side of the own ship shall be identified as the most dangerous ship.

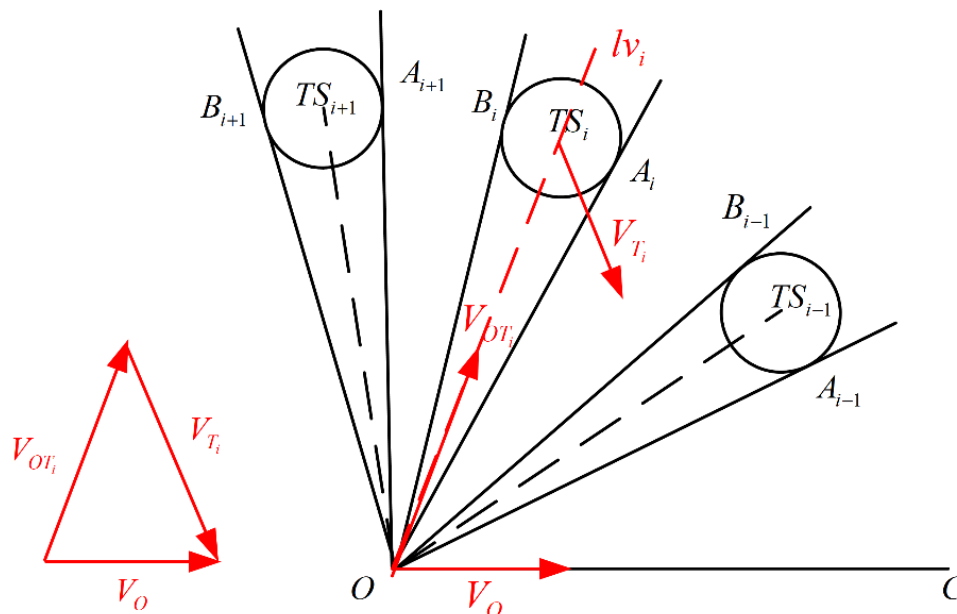


Figure 1 Action set screening algorithm based on velocity obstacle method.

4.3 Action set screening

In order to eliminate the collision risk with the third ship after the own ship adopts a collision avoidance operation to avoid the target ship, the other ships closest to the target ship TS_i should be considered. Judge the velocity obstacle area of the own ship relative to the target ship TS_i and the velocity obstacle area of the ships on both sides of the target ship whether there is an overlap.

The action of collision avoidance is changing course. The optional action range is the range of course. According to the COLREGS, the meeting of ships can be divided into 3 situations: overtaking situation, head-on situation, and crossing situation. Corresponding to the 3 encounter situations, the actions a ship needs to take are shown in **Figure 2**.

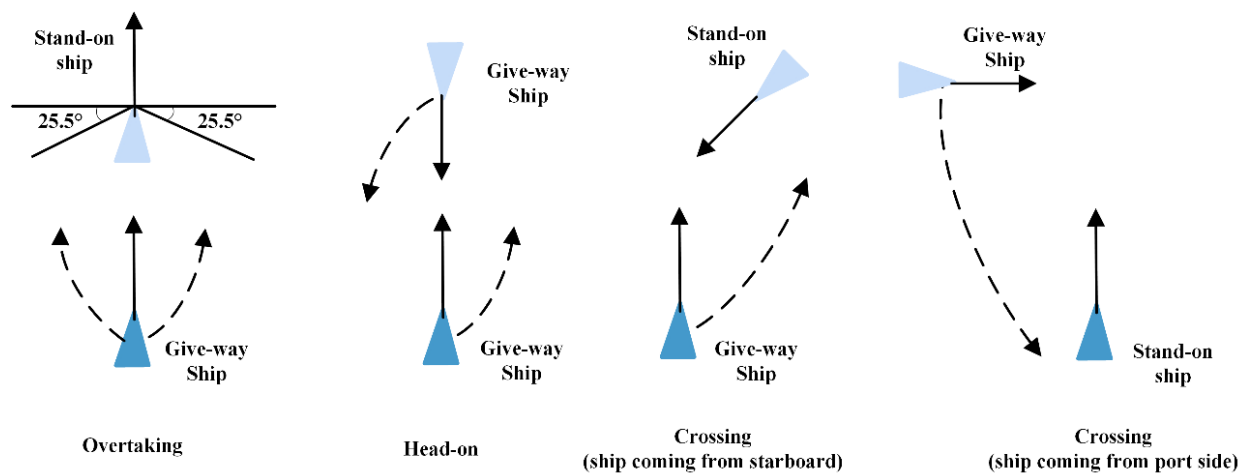


Figure 2 Collision avoidance operations for encounter situations.

The actions of ship collision avoidance in 3 encounter situations (**Figure 2**) are introduced to screen the optional action range of collision avoidance according to the multi-ship encounter situation (**Figure 1**). The optional action range is divided at equal intervals, and actions are selected to form an action set. The rules of action set screening are as follows:

(1) There are dangerous ships on both sides of the target ship TS_i :

1) When only the left velocity obstacle area overlaps or both sides of the velocity obstacle area do not overlap, the optional action range in the relative velocity V_{OT_i} direction are $\angle A_i O B_{i-1}$ (**Figure 1**). Determine the action set for the equal interval partition of $\angle A_i O B_{i-1}$.

2) When only the right velocity obstacle area overlaps or both sides velocity obstacle area overlap, the upper limit of the optional action range in the relative velocity V_{OT_i} direction is $\angle A_{i-1} O C$ (**Figure 1**). Determine the action set for the equal interval partition of $\angle A_{i-1} O C$.

(3) There is only a dangerous ship on the left side of the target ship TS_i . The upper limit of the optional action range in the relative velocity V_{OT_i} direction is $\angle A_i O C$ (**Figure 1**). Determine the action set for the equal interval partition of $\angle A_i O C$.

(4) There is only a dangerous ship on the right side of the target ship TS_i :

1) When right velocity obstacle area overlaps, the upper limit of the optional action range in the relative velocity V_{OT_i} direction is $\angle A_{i-1} O C$ (**Figure 1**). Determine the action set for the equal interval partition of $\angle A_{i-1} O C$.

2) When right velocity obstacle area does not overlap, the optional action range in the relative velocity V_{OT_i} direction is $\angle A_i O B_{i-1}$ (**Figure 1**). Determine the action set for the equal interval partition of $\angle A_i O B_{i-1}$.

5. Solution of optimal action profile

In the situation of multi-ship encounter, each ship selects the ships it needs to avoid and the actions it can take based on the improved extensive game algorithm. The multi-ship collision avoidance game is transformed into a 2-ship collision avoidance game. The solution of the optimal action profile in the process of collision avoidance is essentially to solve the subgame perfect Nash equilibrium. In the game model of ship collision avoidance, the subgame perfect Nash equilibrium refers to the collision avoidance decision profile consisting of the decisions that the 2 ships make. The decision is the most beneficial one for each ship based on another ship's decision. The subgame perfect Nash equilibrium is solved by backward induction and the optimal action profile is obtained.

5.1 Generation of the game tree of ship collision avoidance

Multi-ship collision avoidance is a dynamic game among officers under the situation of multi-ship encounter. When pilots steer the ship, the order is agreed, and the collision avoidance actions are taken under the constraint of COLREGS. Those who take action later know the previous game and the decisions of the former players. The game tree of collision avoidance is shown in **Figure 3**. Suppose the layer of the game tree is m . f_{ij}^N is the state space of layer i , node j , and N is the ship that makes the choice of collision avoidance action at the nodes of this layer. s_{ik}^N is action k that the ship can take when making decisions of collision avoidance at the nodes of layer i . Actions are represented by branches in a game tree. The nodes of layer m are the end nodes, indicating that the game ends here. u_{mj}^N represents the payoffs of the ships at the nodes of layer m .

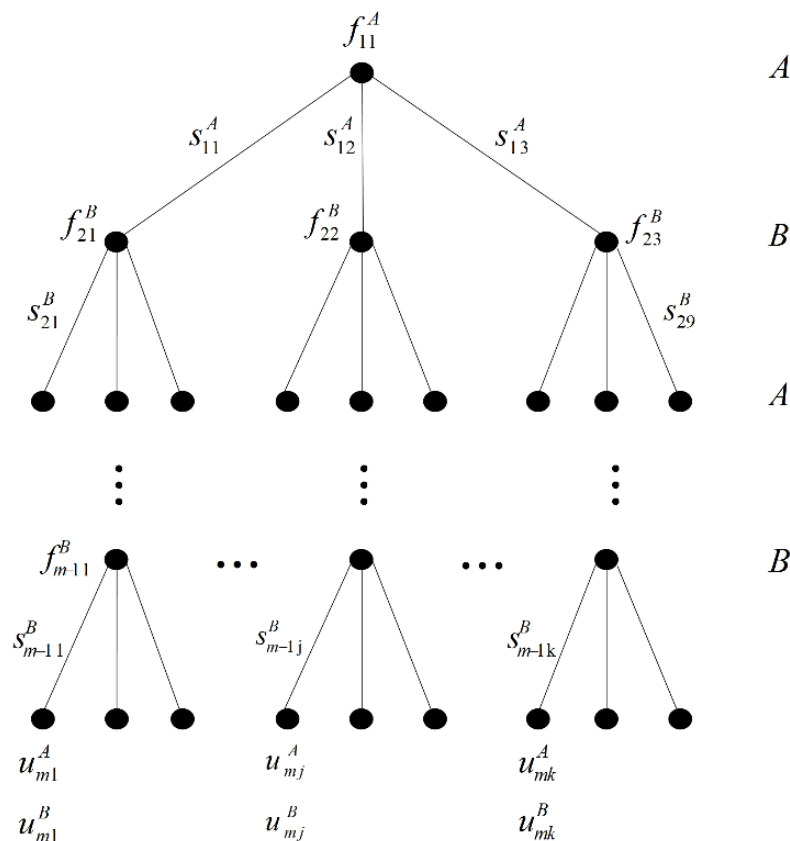


Figure 3 Game tree diagram of collision avoidance between 2 ships.

Starting from the initial state of each ship, data processing, node assignment, and spatial information iteration are carried out to generate the game tree of ship collision avoidance based on the extensive game model of collision avoidance. The main process is as follows:

Step 1: Set and initialize the 2 lists: **OpenList** and **ClosedList**. **OpenList** is used to store the nodes that have been searched to be expanded, and **ClosedList** is used to store the nodes that have been expanded.

Step 2: Initialize the root node of the game tree, set it to the current node, and store it into the **OpenList**.

Step 3: Start expanding the child nodes of the current node, store the child nodes in the **OpenList**, and initialize them. Calculate the new angle of course, current position, and payoff of the 2 ships, then assign the values to the child node. The expanded node is stored in the **ClosedList**.

Step 4: After all the child nodes of the current node are stored into **OpenList**, the first node in **OpenList** becomes the current node and continues to expand its child nodes. Repeat Step 3 to expand the nodes in the **OpenList** in the sequence of insertion until the **OpenList** is empty or the game ends.

The flow of game tree generation algorithm is shown in **Figure 4**.

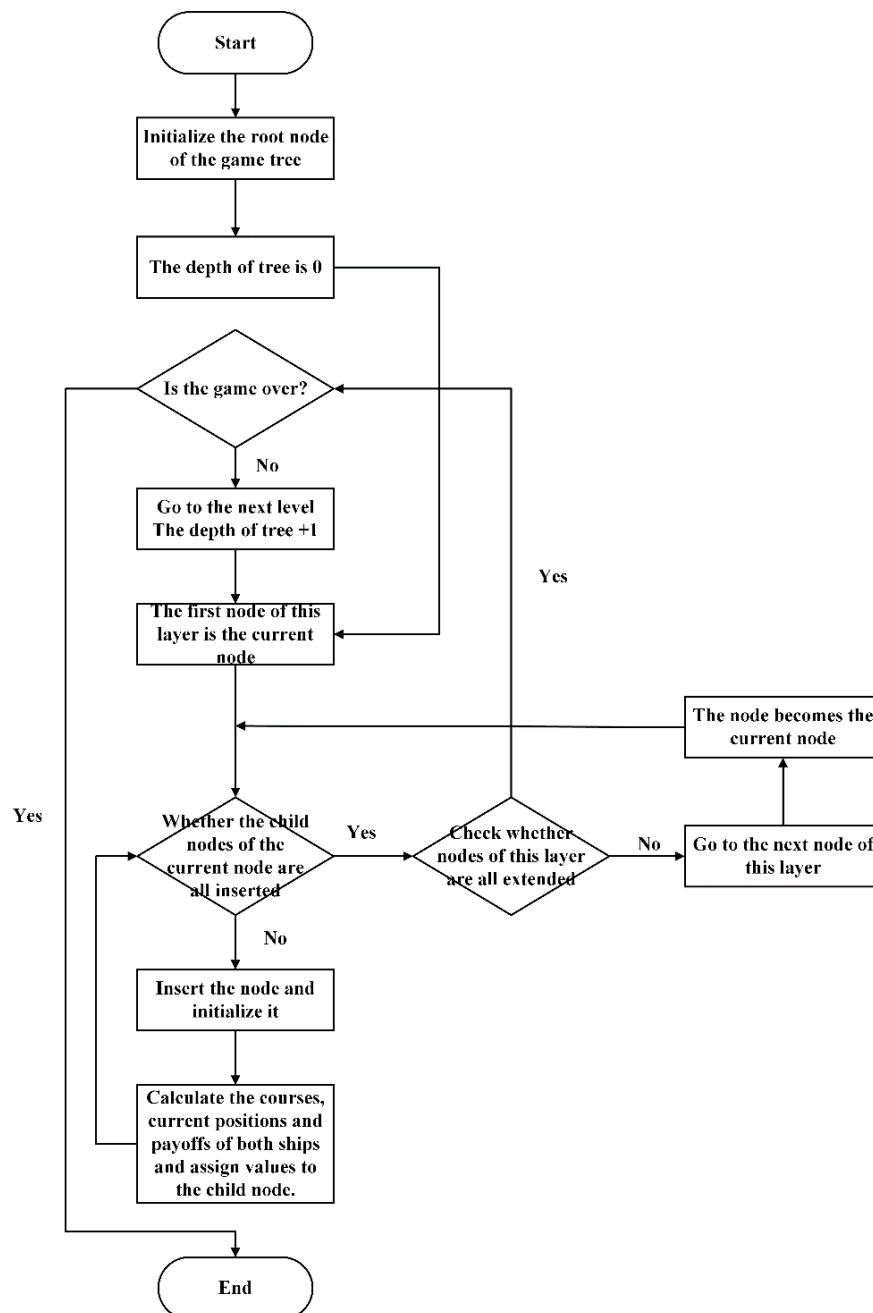


Figure 4 Flow of game tree generation algorithm.

5.2 Backward induction

Backward induction is an effective method to solve the subgame perfect Nash equilibrium. The core idea of backward induction is forward prediction and backward reasoning. The basic process is as follows: starting from the bottom layer, search for action options with the greatest payoff per layer upwards step by step and avoid the non-credible node. The subgame Nash equilibrium is obtained. The steps of using backward induction to solve the optimal action profile are as follows:

Step 1: Search layer $m-1$ and assume that ship B makes the choice of action at the nodes of layer $m-1$.

Step 2: The brother nodes of layer m are compared with each other. Corresponding to each parent node of layer $m-1$, a node is found where ship B has the greatest payoff. The nodes numbers are l_1, l_2, \dots, l_n , where the value of n is equal to the number of nodes in layer $m-1$. The payoffs of ship B are $u_{ml_1}^B, u_{ml_2}^B, \dots, u_{ml_n}^B$ and the payoffs of ship A are $u_{ml_1}^A, u_{ml_2}^A, \dots, u_{ml_n}^A$. The action choices are $s_{m-1l_1}^B, s_{m-1l_2}^B, \dots, s_{m-1l_n}^B$. The payoffs of ship A and ship B at nodes l_1, l_2, \dots, l_n of layer m are taken as the payoffs of ship A and ship B at their parent nodes, and the corresponding action choices are retained.

Step 3: Keep searching one layer up, and ship A is making the decision. The brother nodes of layer $m-1$ are compared with each other. Corresponding to each parent node of layer $m-2$, a node is found where ship A has the greatest payoff. The payoffs of ship A and ship B at these nodes are taken as the payoffs of ship A and ship B at their parent nodes, and the corresponding action choices are retained.

Step 4: Repeat step 2 and step 3 until the search layer is 0.

Step 5: Starting from the root node of the game tree, the previously retained action choices are found layer by layer down. The action profile formed by the action choices is the subgame perfect Nash equilibrium of the game, namely, the optimal action profile solved.

6. Process of improved extensive game method

In the case of a multi-ship encounter situation, each ship takes turns as the own ship. At each moment, each ship performs the following processes:

(1) Collision detection stage. Own ship detects the surrounding environment to determine whether there is a ship that poses a collision risk (Section 4.2). If there is, enter the next stage.

(2) Collision avoidance decision-making stage. First of all, the ship with the highest collision risk is selected as the key avoidance ship of the own ship, and the own ship and the key avoidance ship form a player set. Then, a game order for 2 players is agreed, and the action set of the 2 players is determined based on the velocity obstacle method (Section 4.3). According to the action sets of the 2 ships in the game, the corresponding benefits are calculated, and the game tree of collision avoidance is constructed (Section 5.1). Finally, the backward induction method is used to solve the optimal action profile of the 2 ships (Section 5.2), that is, the course combination of collision avoidance.

(3) Collision avoidance action stage. The course angle of each ship is determined according to the improved extensive game method. Based on incremental PID control, each ship is controlled to avoid collision according to the planned course.

The process is shown in **Figure 5**.

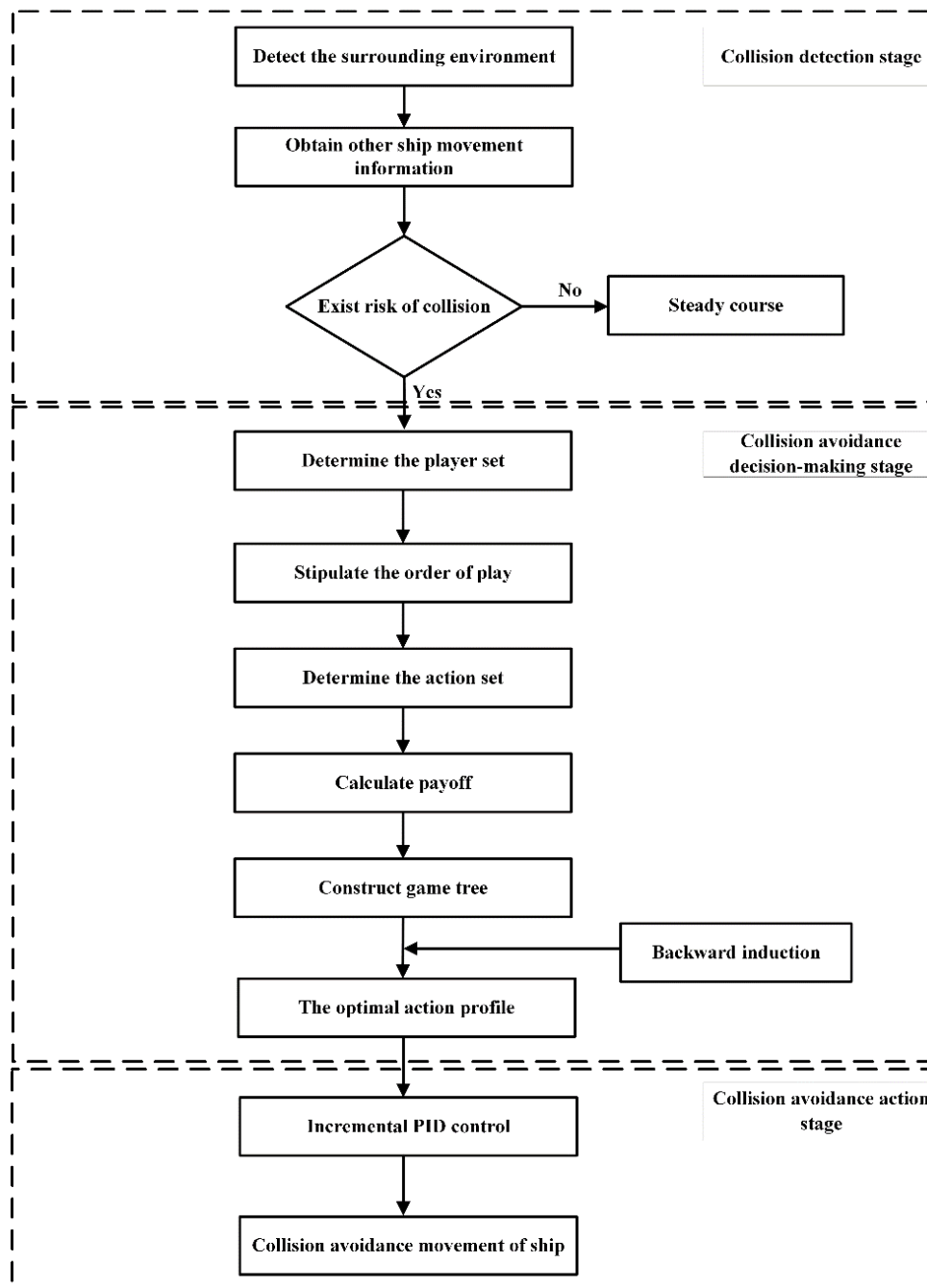


Figure 5 Improved extensive game method multi-ship collision avoidance flow.

7. Multi-ship collision avoidance simulation experiment

7.1 Setup

7.1.1 Simulation experiment ship motion model

In this paper, the ship hydrodynamic model is used as the simulation ship motion model, and the simulation ship motion is modeled based on the MMG model. The parameters of the simulated ship are listed in **Table 1**.

This paper studies an underactuated ship, and the ship motion control mechanism consists of propeller and steering gear. The parameters of the simulated ship are shown in **Table 1**.

Table 1 Main parameters of simulated ship.

Parameter	Value
Ship length overall /m	47.4
Ship breadth /m	8
Draft/m	3.7
Rudder height /m	1.35
Propeller diameter /m	1.56
Design Speed /kn	12
Max rudder angle /(rad)	0.61

7.1.2 Ship domain

The ship domain is modeled as a circle with a radius of 200 m. The formula for the ship domain is as follows:

$$\begin{cases} D_x = x_l + 200 \cos \theta \\ D_y = y_l + 200 \sin \theta \end{cases} \quad (11)$$

7.1.3 Motion control method

Combined with the actual needs and future application, the course control system in this paper is modeled based on the incremental PID control method. The incremental PID control method calculates the control instruction required by the ship turning motion.

The control instruction calculated by the incremental PID control method is the steering angle of the ship. The proportion, integral, and differential parameters of the incremental PID control method are, respectively, set as $K_p = 4$, $K_i = 400$, $K_d = 8$,

$$\begin{aligned} \sigma(T+1) = & \sigma(T) + 4(\psi(T) - \psi(T+1)) + \frac{8(2\psi(T) - \psi(T-1) - \psi(T+1))}{t} + \dots \\ & \dots + 400t(\psi_{goal} - \psi(T+1)) \end{aligned} \quad (12)$$

ψ_{goal} is the ship next time target course.

7.2 2-ship collision avoidance simulation experiment

Before the multi-ship simulation experiment, it is necessary to design 3 encounter situations, such as overtaking, head-on situation, and crossing situation, to verify the effectiveness of the improved extensive game method.

Regulation: if the ship alters course to the starboard side, the course variation is negative; if the ship alters course to the port side, the course variation is positive.

7.2.1 Overtaking situation

A ship is designed to overtake another ship from the stern. The ship being overtaken shall be a stand-on ship, while the ship which overtakes another ship shall be a give-way ship. Ship 1 overtakes ship 2. The initial state of ship 1 and ship 2 are shown in the following **Table 2**.

Table 2 Initial state of ship.

Ship	X (m)	y (m)	ψ (rad)	Speed (kn)
Ship 1	1	120	0	12
Ship 2	1040	120	0	6

The simulation duration is 600 s, and the sampling interval is 1 s. The closest distance between the 2 ships is 202 m, and the maximum steering angle of ship 2 is 0.22 rad to the right. The collision avoidance process in the encounter situation is shown in **Figure 6** below.

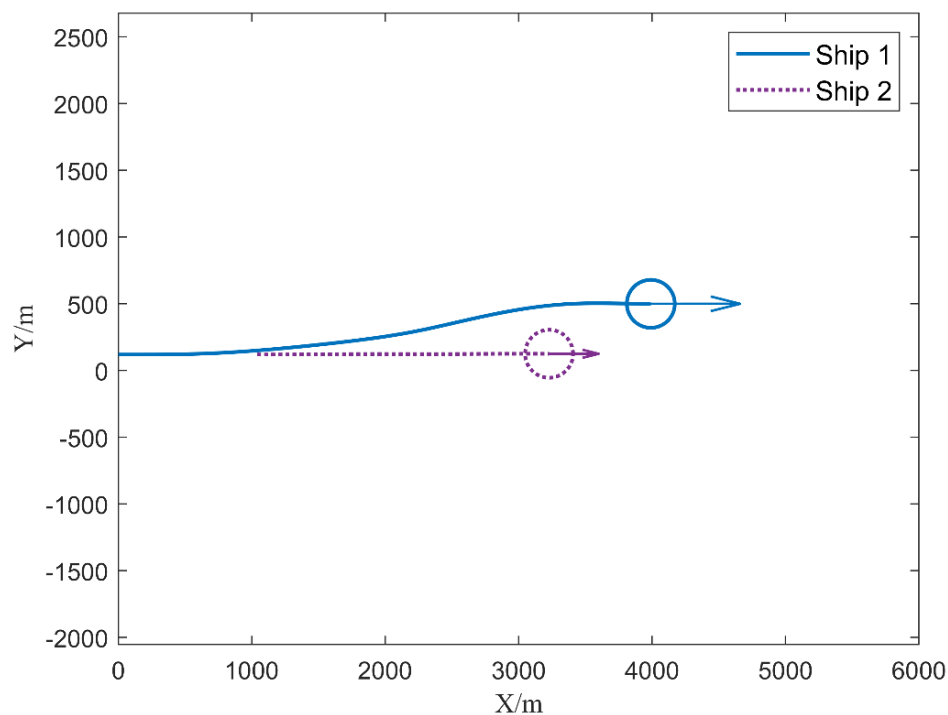


Figure 6 Overtake process of 2 ships.

The course changes for both ships are shown **Figure 7** below.

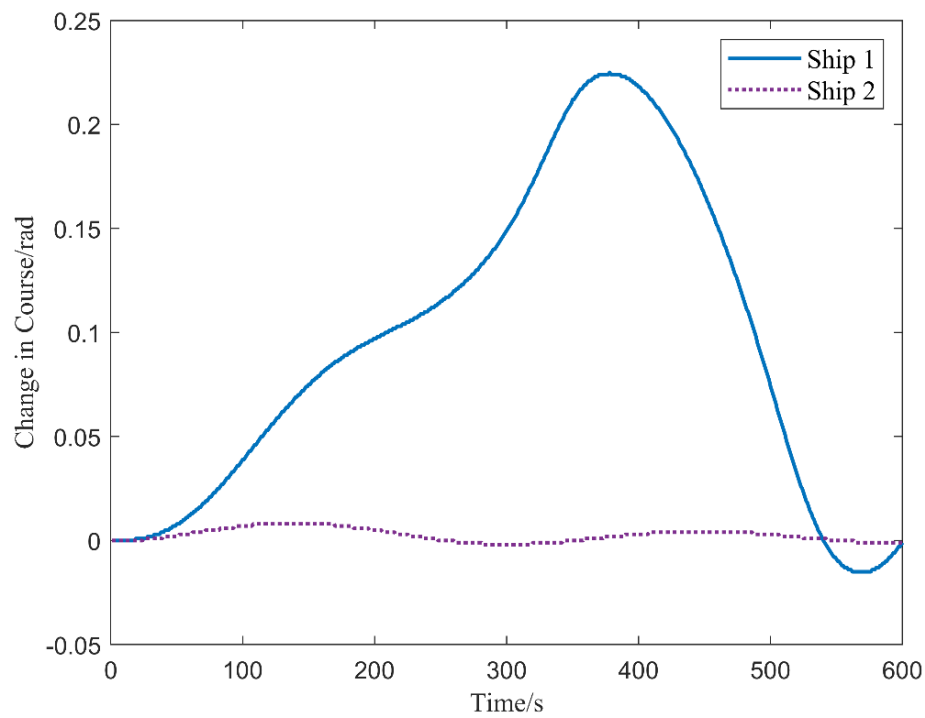


Figure 7 Course change of 2 ships.

The variation trend of distance between 2 ships is shown in **Figure 8** below.

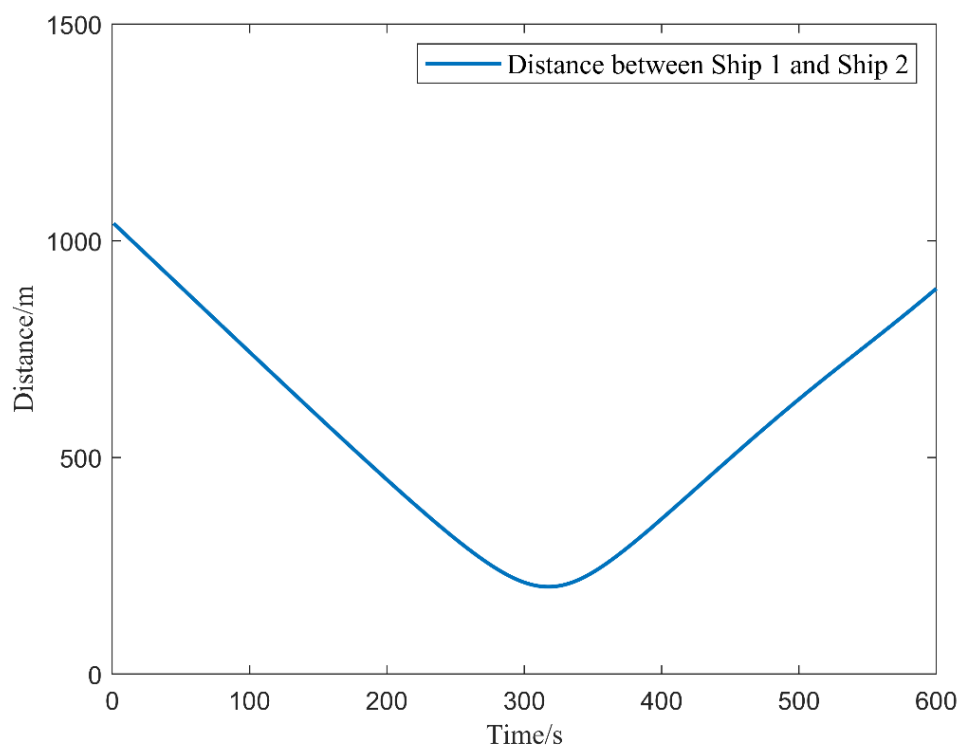


Figure 8 Distance between each ship during simulation.

7.2.2 Head-on situation

A 2 ships head-on encounter situation is designed. The 2 ships in this situation are giving way to each other. Ship 1 sails in the direction of 0 rad and ship 2 sails in the direction of 3.14 rad. The initial states of ship 1 and ship 2 are shown in the following **Table 3**.

Table 3 Initial states of ships.

Ship	x (m)	y (m)	ψ (rad)	Speed (kn)
Ship 1	556	120	0	9
Ship 2	2844	120	3.14	9

The simulation duration is 800 s, and the sampling interval is 1 s. The closest distance between the 2 ships is 332 m. The maximum steering angle of ship 1 and ship 2 are both 0.26 rad to the right. The collision avoidance process in the encounter situation is shown in **Figure 9** below.

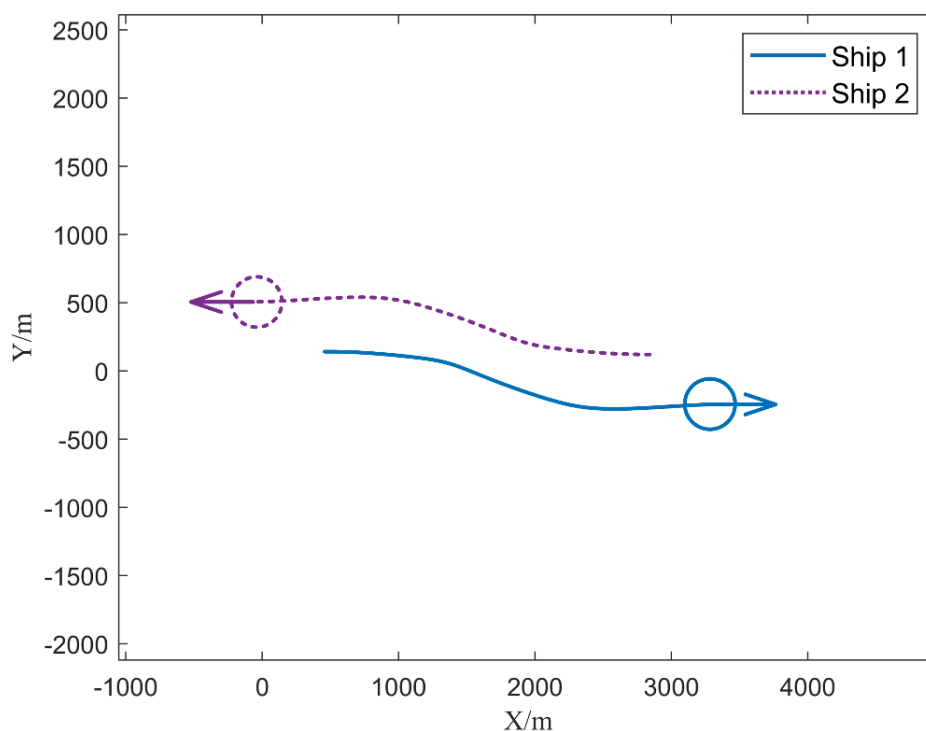


Figure 9 Head-on process of 2 ships.

The course changes for both ships are shown in **Figure 10** below.

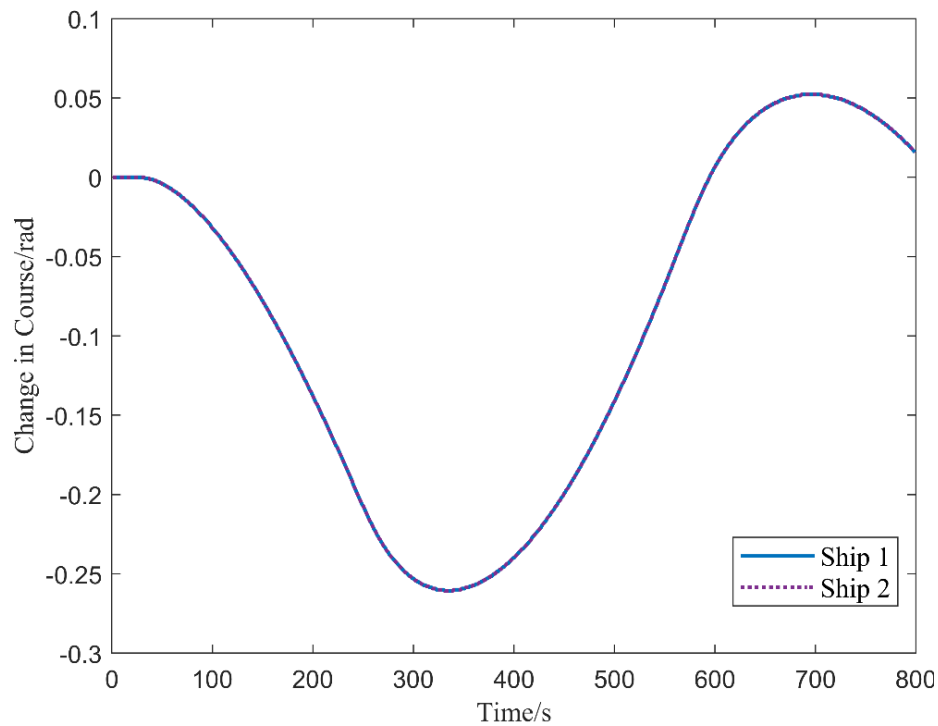


Figure 10 Course change of 2 ships.

The variation trend of distance between 2 ships is shown in **Figure 11** below:

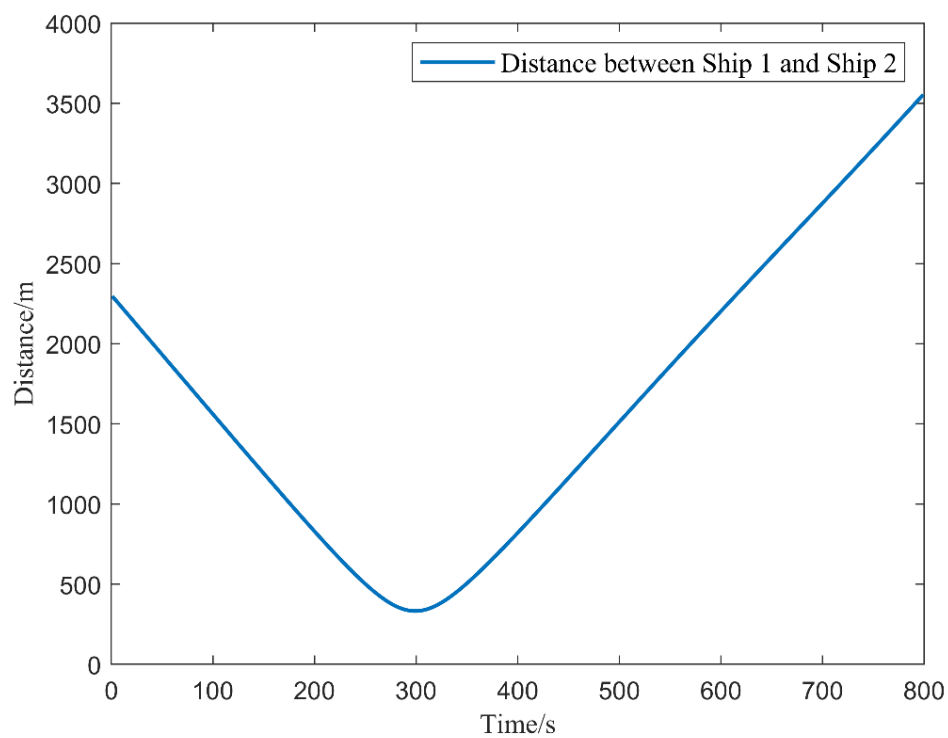


Figure 11 Distance between each ship during simulation.

7.2.3 Crossing situation

A 2 ships crossing situation is designed. Ship 1's initial heading is 0.09 rad, and Ship 2's initial heading is 2.62 rad. Ship 2 comes from the starboard side of ship 1. The initial states of ship 1 and ship 2 are shown in the following **Table 4**.

Table 4 Initial states of ships.

Ship	x (m)	y (m)	ψ (rad)	Speed (kn)
Ship 1	50	120	0.09	9
Ship 2	2047	-301	2.62	9

The simulation duration is 700 s, and the sampling interval is 1 s. The closest distance between the 2 ships is 270 m. The maximum steering angle of ship 1 is 0.45 rad to the right, and the maximum steering angle of ship 2 is 0.31 rad to the right. The collision avoidance process in the encounter situation is shown in **Figure 12** below:

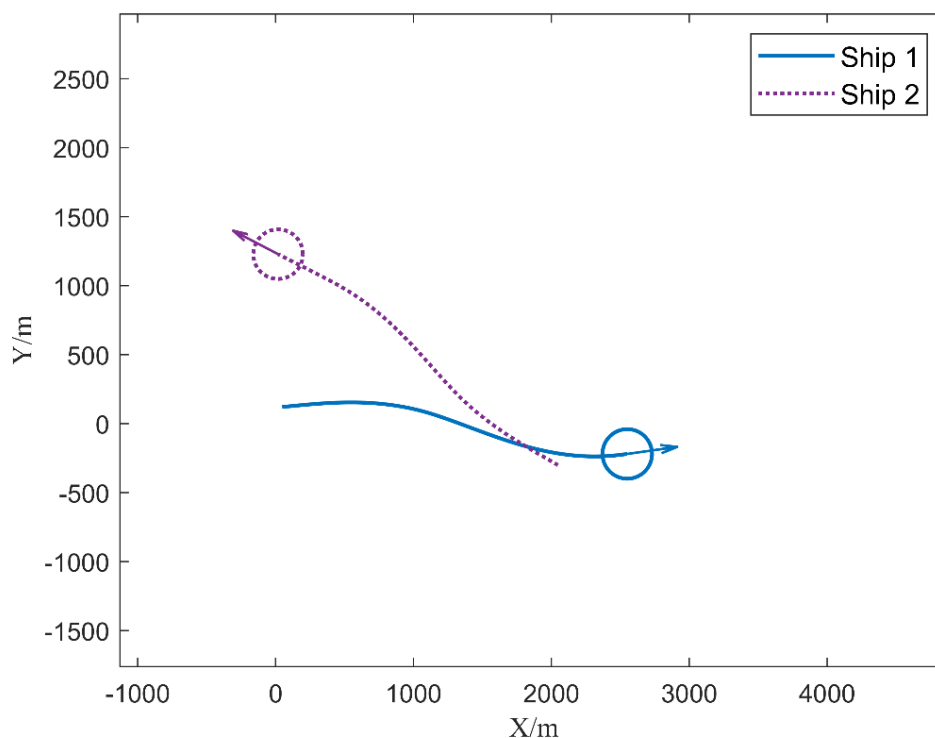


Figure 12 Crossing process of 2 ships.

The course changes for both ships are shown in **Figure 13** below.

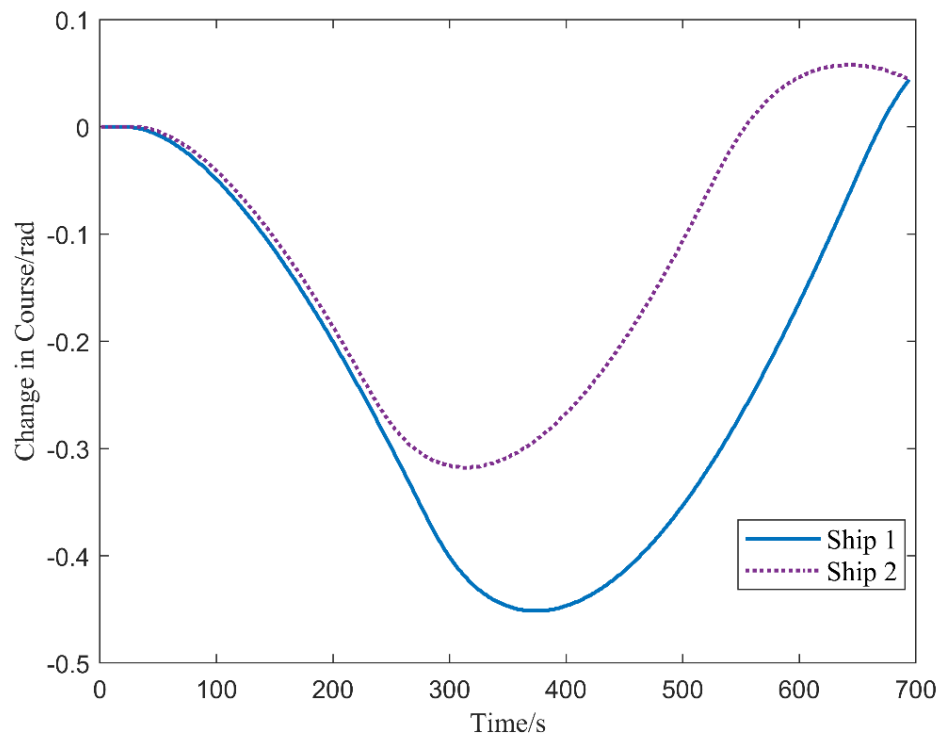


Figure 13 Course change of 2 ships.

The variation trend of distance between the 2 ships is shown in **Figure 14** below.

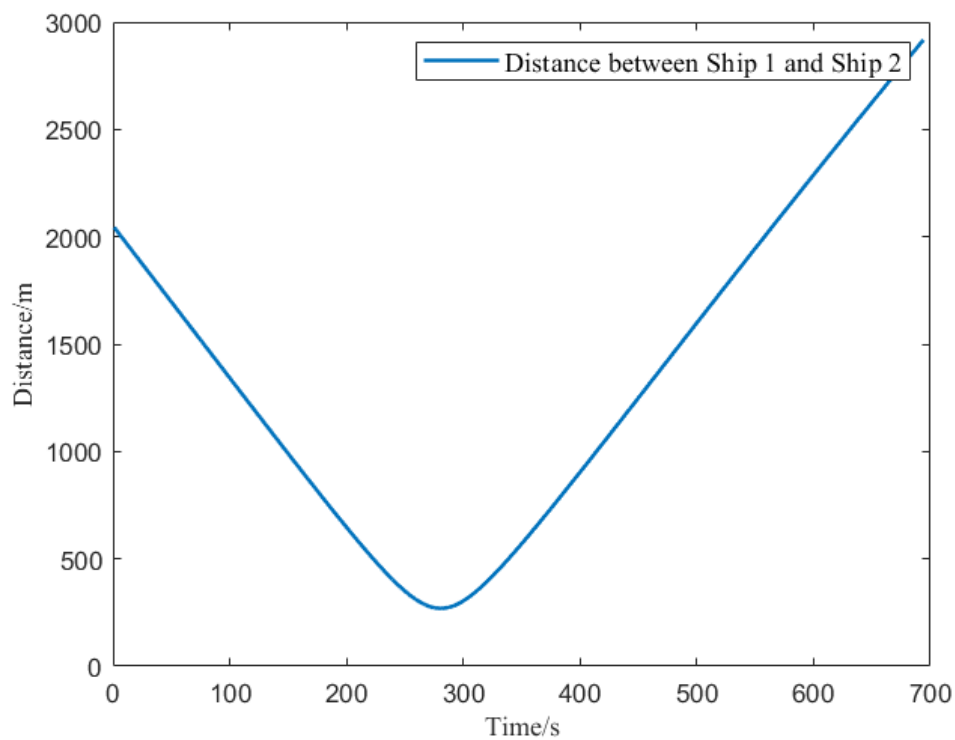


Figure 14 Distance between each ship during simulation.

7.3 Multi-ship collision avoidance simulation experiment

7.3.1 Initial state of the ship

A 4 ship encounter situation experiment scenario is constructed. Ships 1 and 2 sail opposite to ships 3 and 4, and the speed of the 4 ships remains the same. The initial simulation state is shown in the following **Table 5**.

Table 5 Initial state of ships.

Ship	X (m)	Y (m)	ψ (rad)	Speed (kn)
Ship 1	1,800	120	0	9
Ship 2	3,547	120	0.0949	9
Ship 3	7,844	596.5	3.14	9
Ship 4	9,047	-301.6	3	9

7.3.2 Collision avoidance simulation analysis

The simulation duration is 1,800 s, and the sampling interval is 1 s. Four representative moments, 0, 500, 850, and 1,800 s, are selected to demonstrate the collision avoidance decision of the improved extensive game method proposed in this paper. The situation of collision avoidance of ships at each time is shown in **Figure 15**, and detailed data are listed in **Table 6**.

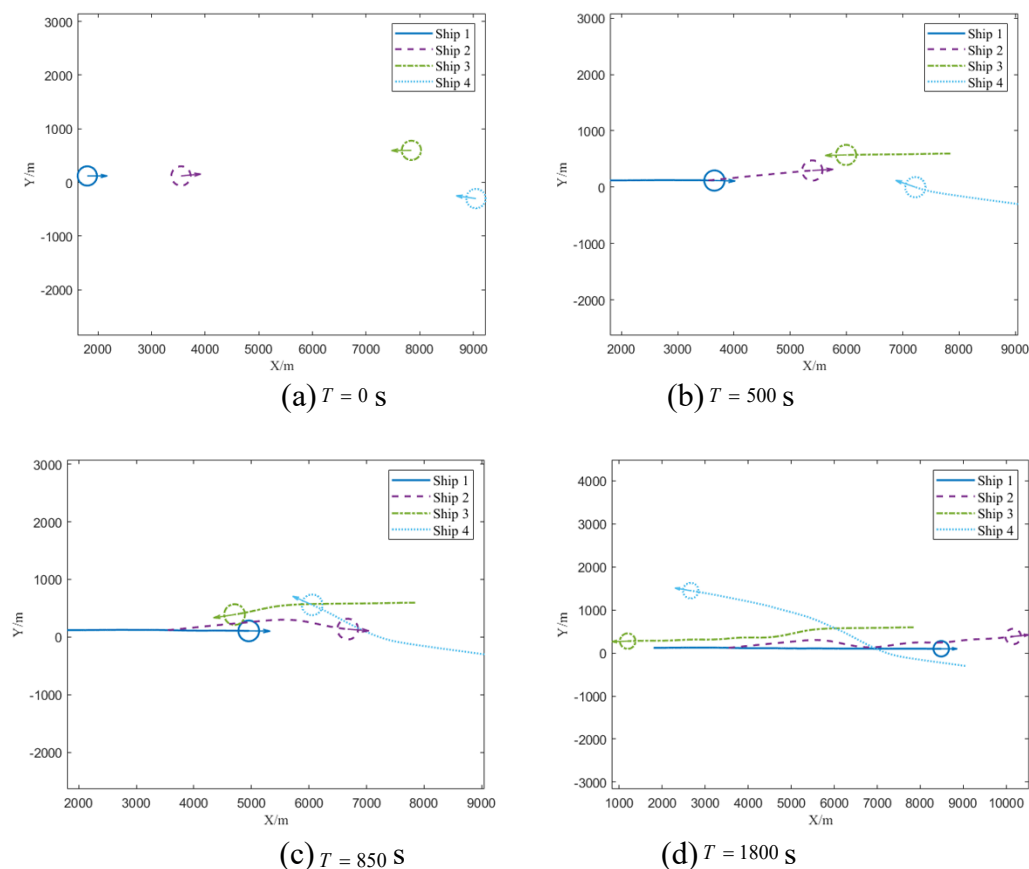


Figure 15 Simulation results of multi-ship collision avoidance.

Detailed data of ship motion at selected representative moments are shown in the following table.

Table 6 Simulation data of multi-ship collision avoidance process.

Time (s)	Ship	X (m)	y (m)	ψ (rad)
0	Ship 1	1,800	120	0
	Ship 2	3,547	120	0.0949
	Ship 3	7,844	596.5	3.14
	Ship 4	9,047	-301.6	3
500	Ship 1	3,656	117.4	0
	Ship 2	5,395	295.6	0.056
	Ship 3	5,988	571.6	3.166
	Ship 4	7,219	1.052	2.811
850	Ship 1	4,958	106.5	6.270
	Ship 2	6,677	137.3	6.201
	Ship 3	4,704	389.6	3.295
	Ship 4	6,052	560.2	2.724
1,800	Ship 1	8,487	97.88	0
	Ship 2	10,160	383.4	0.105
	Ship 3	1,196	275.2	3.174
	Ship 4	2,661	1,447	2.975

The course change curves of each ship are drawn in **Figure 16**. The distance change curves of each ship in the multi-ship encounter situation are shown in **Figure 17**. The performance of the improved extensive game method proposed in this paper is analyzed.

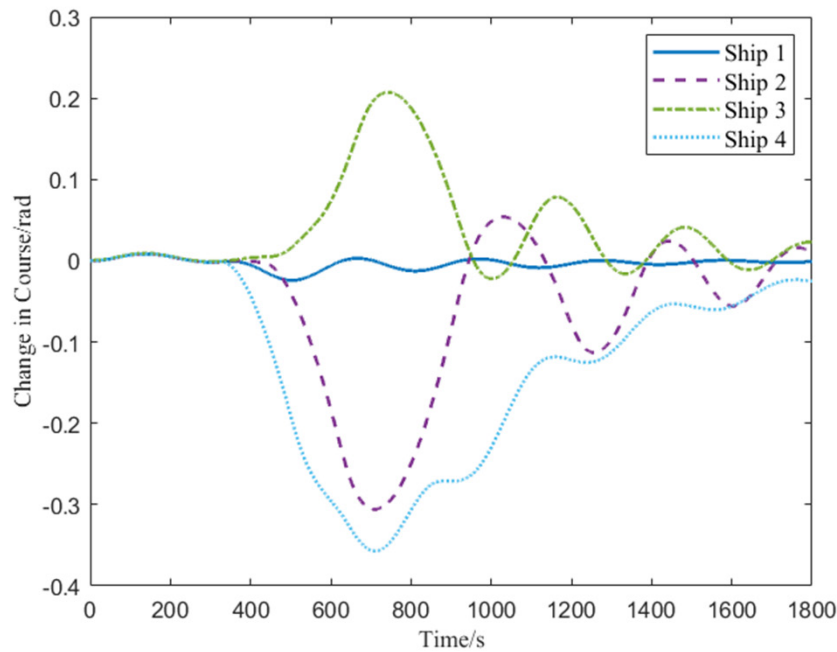


Figure 16 Course change of each ship.

17. The distances between each ship during the simulation are calculated, as shown in **Figure 17**.

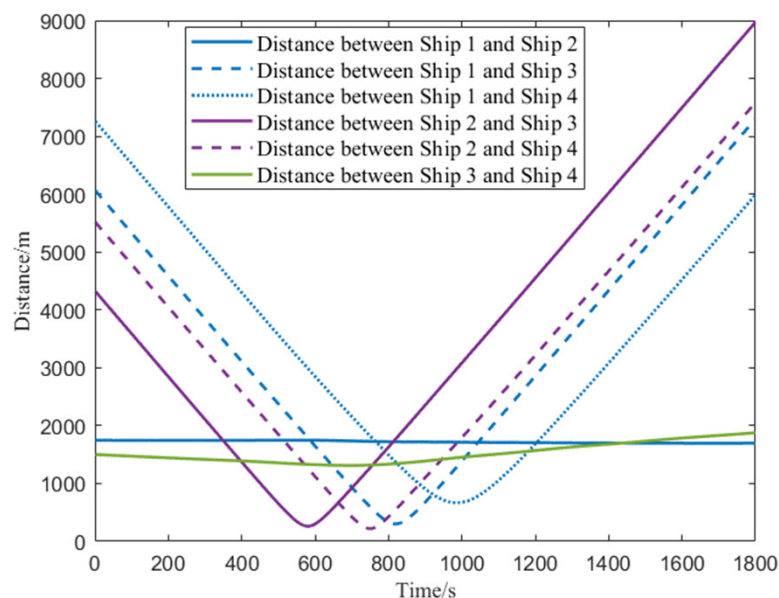


Figure 17 Distances between each ship during simulation.

Combined with **Figures 15 - 17**, the collision avoidance decision of the improved extensive game method is analyzed:

Ship 1 keeps almost straight from its initial position.

Ship 2 detects ship 4 approaching her starboard at 450s, ship 2 forming a crossing situation with ship 4, and ship 2 and ship 4 showing the existence of a dangerous collision risk. In accordance

with the COLREGS, the ship which has the other on her own starboard side shall keep out of the way. At 450s, ship 2 is turning right 0.304 rad. At 744 s, ship 2 completes the turning operation and turns to the target course. At this time, the nearest distance between ship 2 and ship 4 is 223.7 m, which is larger than the design value in the ship domain. The relative velocity of ship 2 and ship 4 does not intersect the ship domain of ship 4, and collision avoidance is completed. Ship 2 resumes course after passing ship 4 safely.

Ship 3 sails on an initial course of 3.14 rad; at 300 s, ship 3 forms a head-on situation with ship 2. Based on the velocity obstacle method, the improved extensive game method determines that there is no collision risk. Ship 3 and ship 2 continue to sail on their initial courses. At 580 s, ship 3 passes ship 2 safely. After 580 s, in order to better test the effectiveness of the improved extensive game method, ship 3's target course is changed to 3.35 rad to make ship 3 turn left at 0.21 rad. Ship 3 completes the turning operation and turns to the target course at 749 s, posing a collision risk situation with Ship 1. In the crossing situation posed by ship 3 and ship 1, the improved extensive game method calculates that the avoidance action needed to be taken by ship 3 is turning right at 0.21 rad. At 821 s, the course of ship 3 is 3.31 rad, and the nearest distance between ship 3 and ship 1 is 302.4m, which is larger than the design value of ship domain. The relative velocity of ship 3 and ship 1 does not intersect the ship domain of ship 1, and collision avoidance is completed. Ship 3 resumes course after passing ship 1 safely.

Ship 4 sails on initial course at 3.0 rad. At 340 s, the relative velocity of ship 4 and ship 1,2 intersect the ship domain of ship 1,2. Ship 1, ship 2, and ship 3 pose a collision risk situation. The risk value is sorted according to the method proposed in Section 3.2. The collision risk between ship 4 and ship 2 is the highest, and ship 4 needs to avoid ship 2. Ship 4 is sailing on the port side of ship 2 relative to ship 2. Therefore, in order to ensure maximum payoff, ship 4 needs to turn right to avoid collision, and its turn right will not pose a collision risk with ship 1. Detailed analysis of ship 4 avoidance of ship 1 and 2 can be found in Section 7.4.1. At 340 s, the improved extensive game method calculates that the avoidance action needed to be taken by ship 4 is turning right at 0.375 rad. At 744 s, ship 4 completes the turning operation and turns to the target course. The nearest distance between ship 4 and ship 2 is 223.7 m, which is larger than the design value of ship domain. The relative velocity of ship 4 and ship 2 does not intersect the ship domain of ship 2, and collision avoidance is completed (**Table 7**).

Table 7 Collision avoidance information of ships.

	Most dangerous ship	Decision-making of collision avoidance (rad)	Nearest distance (m)
Ship 1	Ship 3	0	302.4
Ship 2	Ship 4	0.304	223.7
Ship 3	Ship 1	0.210	302.4
Ship 4	Ship 2	0.357	223.7

7.4 Performance analysis

7.4.1 Action set screening comparison analysis

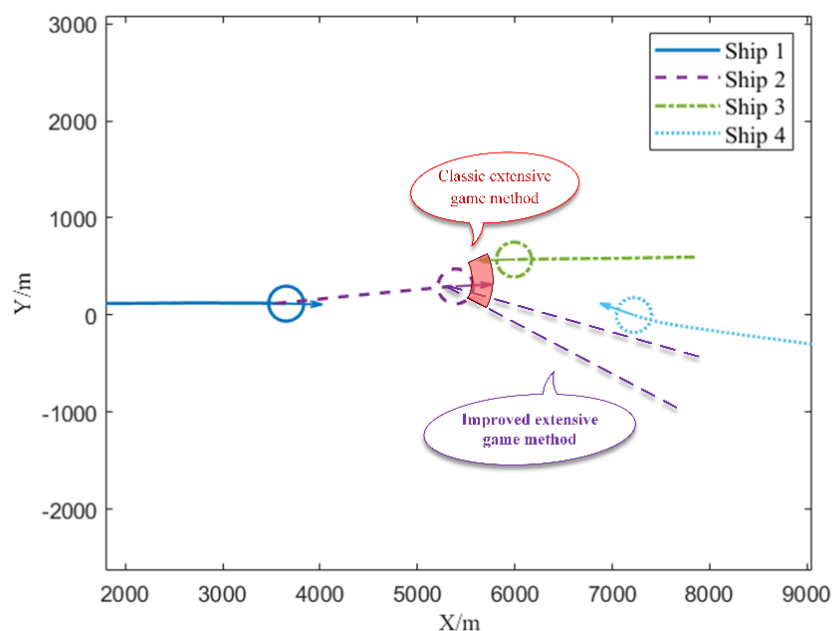
The improved extensive game method screens the optional action range based on the velocity obstacle method, and the classic extensive game method screens the optional action range based on experience. In order to demonstrate in detail the collision avoidance decision of the

method proposed in this paper, the multi-ship encounter situation constituted by ships at 500 s is selected for analysis. At 500 s, ship 2 and ship 4 need to avoid each other.

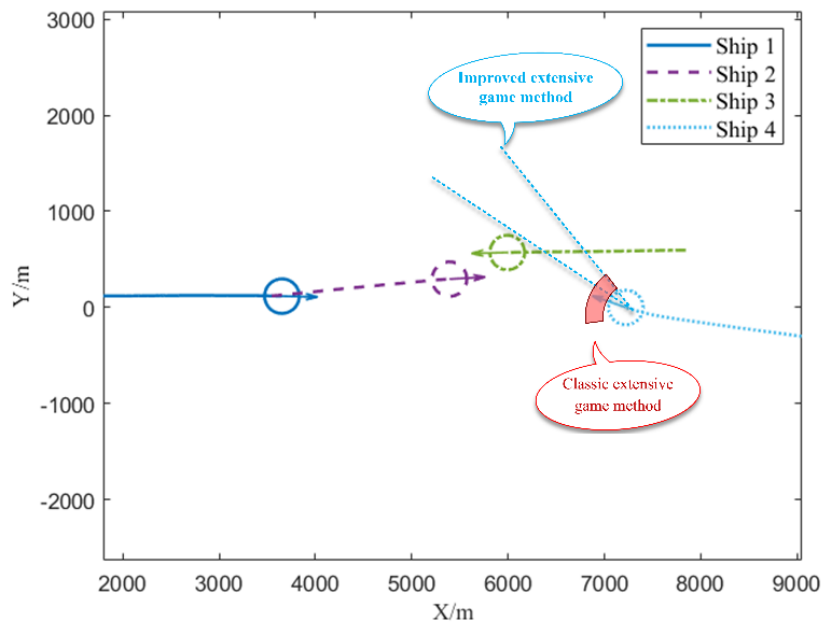
The improved extensive game method screens the action set based on the velocity obstacle method and can avoid the collision danger with the third ship after the own ship adopts collision avoidance operation to avoid the target ship. For ship 4, ship 2 domain overlaps with ship 1 and ship 3. Ship 4 is sailing on the port side of ship 2 relative to ship 2. Therefore, in order to ensure maximum benefits, ship 4 needs to turn right to avoid collision. According to the velocity obstacle method, the optional action range of ship 4 to avoid ship 2 is the right area of ship 3 (as shown in **Figure 18(b)**). The action set based on optional action range of ship 4 to avoid the secondary collision risk between ship 1 and ship 3 is determined.

According to the steering limit of the rudder angle (**Table 1**), the optional action range of ship 2 is the left area of ship 4 (**Figure 18(a)**), ranging from 5.809 rad to 6.232 rad, and the optional action range span is 0.423 rad. The optional action range of ship 4 is the left area of ship 2 (**Figure 18(b)**), ranging from 2.181 rad to 2.547 rad, and the optional action range span is 0.366 rad. Divide the optional action range into 5 equal parts to determine the ship action set. The course interval between actions in ship 2 action set and ship 4 action set are 0.085 rad and 0.073 rad, respectively. Because the feasible collision avoidance area of each ship is determined based on the velocity obstacle method, any course in the action set of each ship will make the ship pass her dangerous ship safely.

The classic extensive game method determines the ship collision avoidance action set based on experience. Generally, the ship collision avoidance action set is determined within the range of 0.52 rad (30 °) on the port and starboard side of ship, as per the red sector shown in **Figures 18(a)** and **18(b)**. Based on this optional action range, a game tree of 2 ships collision avoidance is constructed to solve the optimal action profile. The optional action range based on experience screening has a large span, which is 1.04 rad. The optional action range of the 2 ships is divided into 5 equal parts to determine the action set. The interval between each action is about 0.29 rad.



(a) Optional action range of ship 2



(b) Optional action range of ship 4

Figure 18 Comparison of optional action range when $T = 500$ s.

A comparison of the 2 methods in the screening action sets is shown in **Table 8**.

Table 8 Comparison of action set screening.

	Ship number	Optional action range (rad)	Course interval (rad)
Improved extensive game method	Ship 2	0.423	0.085
	Ship 4	0.366	0.073
Classic extensive game method	Ship 2	1.04	0.29
	Ship 4	1.04	0.29

According to the analysis of **Table 8**, the optional action range of collision avoidance screened by the improved extensive game method is reduced by 62.08 %, and the action precision of the action set is improved by 72.76 %.

According to the payoff function established in this paper, the payoff corresponding to the action of each ship in the action set is calculated. The smaller the offset, the larger the payoff. The collision avoidance game tree of ship 2 and 4 is constructed and the optimal action profile is determined by backward induction. At 500 s, for improved extensive game method, the action set and offset of ship avoidance actions are as listed in **Table 9**.

Table 9 Table of action set and offset.

	Change in Course (rad)	Payoff (m)
Ship 2	-0.1925	-0.7143
	-0.277	-1.0262
	-0.3615	-1.3362
	-0.446	-1.6439
	-0.5305	-1.9486
Ship 4	-0.337	-1.244
	-0.410	-1.511
	-0.483	-1.776
	-0.557	-2.038
	-0.630	-2.298

According to the analysis in **Table 9**, collision avoidance actions of ship 2 and ship 4 at 500 s are turning right at 0.19 rad and turning right at 0.33rad. For the classic extensive game method, if the accuracy of course interval is improved by increasing the number of actions, the number of nodes in the game tree will increase, and the time required to solve collision avoidance actions will also increase.

7.4.2 Comparative analysis of computing resource consumption

For the collision avoidance decision of 2 ships, this section proves the effectiveness of the improved extensive game method to increase decision-making speed by comparing the CPU time with the classic extensive game method.

Under the condition of the same course angle interval, this section only calculates the efficiency of the 2 methods using backward induction to solve the optimal action profile. For the classic extensive game method, in order to keep the interval of action in the action set the same as the improved extensive game method, the number of actions in the action set should be increased to 13.

In the case of the same action interval, based on the game tree constructed by the 2 methods, the optimal action profile is solved 1,800 times. The experiment platform is a portable mobile computer based on Intel Core CPU(i7-9750H-2.60GHz). The consumed CPU time is recorded. **Figure 19** represents the percentage of the time consumed by the proposed method in the time consumed by the classic method.

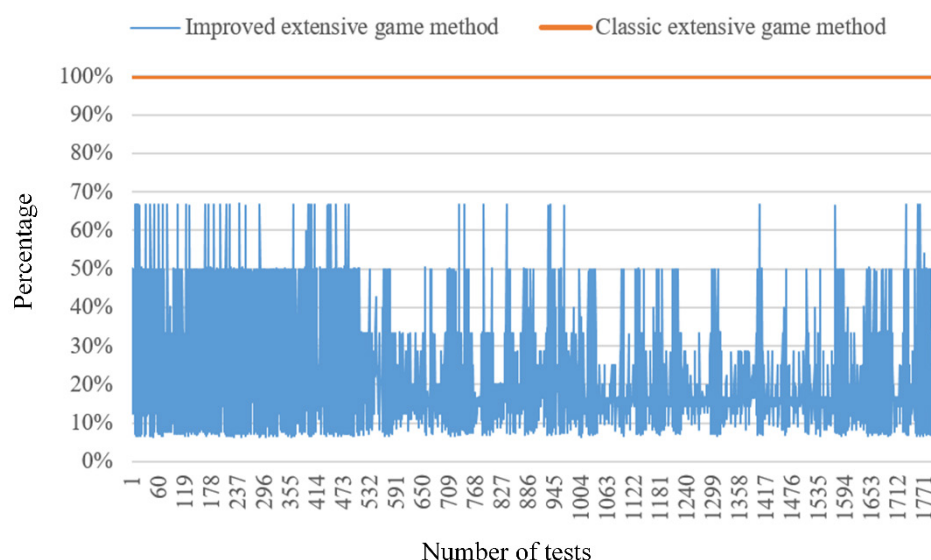


Figure 19 CPU time comparison.

With the same accuracy of actions, the CPU time of the classic extensive game method to solve the optimal action profile is about 5 times that of the improved extensive game method. Therefore, the improved extensive game method consumes less time and improves the decision-making efficiency of collision avoidance.

8 Conclusion

The method proposed in this paper, based on the velocity obstacle method, determines the most dangerous ship that each ship needs to avoid, and the action set that each ship needs to take. The feasible region of velocity is taken as the optional action range of the game to narrow down the action set. The proposed method can eliminate the collision risk with the third ship after the own ship adopts a collision avoidance operation to avoid the target ship. There are increased screening speed for dangerous ships and collision avoidance actions. The method proposed in this paper is beneficial to improve the decision-making efficiency of collision avoidance action.

Simulation results show that the range of feasible collision avoidance areas screened by the improved extensive game method is reduced by 62.08 % compared with the classic extensive game method, and the accuracy of actions in action set is increased by 72.76 % under the premise of ensuring the navigation safety of all ships in the encounter situation. With the same accuracy of actions, the CPU time of the classic extensive game algorithm to solve the optimal action profile is about 5 times that of the improved extensive game algorithm.

There are still many deficiencies in this study, which can be further improved in the following aspects: (1) this paper only considers the single collision avoidance mode of steering. Other collision avoidance modes, such as slowing down and stopping engines, can be considered in subsequent studies. (2) The heading control method of collision avoidance maneuver in this paper is the incremental PID control method. In the future, the PID heading control method can be improved by combining it with a deep learning algorithm to achieve ship motion control with stronger robustness. (3) In the future, researches on ship collision avoidance game under the condition of asymmetric information can be carried out to solve some situations in which ships do not cooperate in collision avoidance.

Acknowledgment

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