Testing the Ratio of the Coefficients of Variation for the Inverse Gamma Distributions with an Application to Rainfall Dispersion in Thailand

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Abstract

In Thailand, droughts are regular natural disasters that happen nearly every year due to several factors such as precipitation deficiency, human activity, and global warming. Since annual rainfall amounts fit an inverse gamma (IG) distribution, we consider testing annual rainfall dispersion via the ratio of the coefficients of variation (CVs). Herein, we present three statistics for testing the ratio of the CVs of the IG distributions based on the fiducial quantities (FQ) and the Bayesian methods by the Jeffreys and uniform priors. We evaluated their performances by using Monte Carlo simulations conducted under several shape parameter values for the IG distributions based on empirical type I error rates and powers of the tests. The simulation results reveal that the empirical type I error rates of all test statistics were close to the nominal significance level of 0.05 for all situations. In the case of the power of the test, the test statistics based on the Bayesian method by the Jeffreys prior performed better than other test statistics for equal sample sizes. In case of unequal sample sizes, the test statistics based on the Bayesian method by the Jeffreys and uniform priors performed well which based on the hypothesized values of ratio of the CVs. Furthermore, the efficacies of the proposed test statistics were illustrated by applying them to annual rainfall dispersion in Buriram and Chaiyaphum ,Thailand.

Keywords: Statistical Test, Measure of Dispersion, Skew Distribution, Simulation, Meteorology

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การทดสอบอัตราส่วนของสัมประสิทธิ์การแปรผันของการแจกแจงแกมมาผกผันกับการประยุกต์ใช้กับการกระจายของปริมาณน้ำฝนในประเทศไทย

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บทคัดย่อ

ความแห้งแล้งเป็นภัยธรรมชาติที่เกิดขึ้นเกือบทุกปีของประเทศไทย เนื่องจากปัจจัยหลายประการ เช่น การขาดแคลนน้ำฝน กิจกรรมของมนุษย์ และภาวะโลกร้อน ผู้วิจัยสนใจการทดสอบการกระจายของปริมาณน้ำฝนรายปีโดยพิจารณาจากอัตราส่วนของสัมประสิทธิ์การแปรผัน เนื่องจากปริมาณน้ำฝนรายปีมีการแจกแจงแกมมาผกผัน ในงานวิจัยนี้ได้ใช้ตัวสถิติทดสอบ 3 ตัว เพื่อทดสอบอัตราส่วนของสัมประสิทธิ์การแปรผันของการแจกแจงแกมมาผกผัน โดยอิง Fiducial quantities (FQ) วิธีของเบส์โดยใช้การแจกแจงก่อน Jeffreys และวิธีของเบส์โดยใช้การแจกแจงก่อนเอกรูป สำหรับการประเมินประสิทธิภาพของตัวสถิติทดสอบจะใช้การจำลองแบบมอนติคาร์โล โดยจำลองข้อมูลให้มีการแจกแจงแกมมาผกผันภายใต้พารามิเตอร์บ่งรูปร่างหลายค่า และพิจารณาอัตราความผิดพลาดประเภทที่ 1 เชิงประจักษ์ และกำลังการทดสอบเชิงประจักษ์ ผลการจำลองแสดงให้เห็นว่าอัตราความผิดพลาดประเภทที่ 1 เชิงประจักษ์ของตัวสถิติทดสอบทุกตัวมีค่าเข้าใกล้ระดับนัยสำคัญ 0.05 ในทุกสถานการณ์ เมื่อพิจารณากำลังการทดสอบเชิงประจักษ์ ตัวสถิติทดสอบอิงวิธีของเบส์โดยใช้การแจกแจงก่อน Jeffreys ให้กำลังการทดสอบเชิงประจักษ์มากกว่ากำลังการทดสอบเชิงประจักษ์ของตัวสถิติอื่น ๆ ในกรณีที่ขนาดตัวอย่างเท่ากัน ส่วนในกรณีที่ขนาดตัวอย่างไม่เท่ากัน ตัวสถิติทดสอบอิงวิธีของเบส์โดยใช้การแจกแจงก่อน Jeffreys และการแจกแจงก่อนเอกรูป มีประสิทธิภาพดีซึ่งขึ้นอยู่กับค่าอัตราส่วนของสัมประสิทธิ์การแปรผันที่ทำการทดสอบ นอกจากนั้น ประสิทธิภาพของตัวสถิติทดสอบที่นำเสนอได้แสดงให้เห็นโดยนำไปประยุกต์ใช้กับการกระจายของปริมาณน้ำฝนรายปีในจังหวัดบุรีรัมย์และชัยภูมิ

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Introduction

Since damage from natural disasters has increased due to anomalous global climate changes, researchers have become interested in studying their occurrences. Thailand has been divided into six geographical regions by the National Research Council: north, northeast, central, east, west, and south; many of them are prone to droughts but they most often occur in the central northeastern part of Thailand. Drought in Thailand directly affects agriculture and water resources, which has a significant impact on the country's economy since most of the country is agrarian.

Buriram and Chaiyaphum, two provinces of the northeastern in Thailand, are faced with drought every year due to long periods of little rain causing a severe shortage of water for both consumption and farming (Srichaiwong et al., 2020). In June 2015, Farmers in Buriram pleaded for government assistance as more than 50% of their farmlands were attacked by the worst drought in 50 years (Pattayamail, 2022a). Moreover, in July 2019, parts of Chaiyaphum were faced with a severe drought, and the water volume in the Chulabhorn Dam decreased to its lowest level in 30 years (only 25% of its capacity) (Pattayamail, 2022b). Additionally, in January 2020, eight hospitals in Chaiyaphum were impacted by the drought, leading to the Chaiyaphum Provincial Public Health Office drilling artesian wells to reserve water for medical services and sufficient staff consumption for at least three days while also requesting citizens to help by saving water (Nationthailand, 2022).

Whenever there are long periods of rainfall deficiency for one season or more, droughts take place (Eartheclipse, 2022). The coefficient of variation (CV) can be applied to explain rainfall dispersion in different regions since the rainfall amount varies vastly depending on the region and season. The CV is a unit-free statistical measure of variability relative to the population mean (Albatineh et al., 2017). The standard formula for the CV is expressed as $\theta = \sigma / \mu, \mu \neq 0$ where σ and μ represent the population standard deviation and population mean, respectively. The CV is a useful statistic for comparison in two or more data sets with different units. The estimation of the CV has been extensively used in several fields of science, medicine, engineering, business, economics and others (see Nairy and Rao, 2003). For example, Faber and Korn (1991) discussed the usage of the CV method for analyzing synaptic plasticity. The CV were used to measure the spatial and temporal correlation of global solar radiation (Calif and Soubdhan, 2016). Reed et al. (2002) applied the CV for assessing the variability of quantitative assays. The CV were applied to index relative internal variability of work groups on numerous dimensions (Bedeian and Mossholder, 2000). The CV for monitoring variability in statistical process control was discussed by Kang et al. (2007). Castagliola et al. (2011) developed a new approach to monitor the CV by two one-sided exponentially weighted moving average charts of the CV squared. The methodology for adjusting the standard CV to account for the systematic dependence of population variance from the population mean was discussed by Döring and Reckling (2018).

In probability and statistics, the inverse gamma (IG) distribution is a two-parameter continuous distribution on the positive real line. It is the distribution of the reciprocal of a random variable distributed according to the gamma distribution (Abid and Al-Hassany, 2016). The IG distribution is most often used as a conjugate prior distribution in Bayesian statistics. There are several research works to study the distribution of the IG. For example, IG distribution is used as the prior distributions for variance parameters in hierarchical models (Gelman, 2006). Abid and Al-Hassany (2016) studied some points related to the IG distribution. The estimation methods based on the method of moments, maximum likelihood, and Bayesian methodology to estimate the parameters of an IG distribution were discussed by Llera and Beckmann (2016). Glen and Leemis (2017) applied the IG distribution to survival studies.

The review literature on testing the ratio of the CVs for the IG distributions is limited. However, there are several approaches available for calculating the confidence intervals for the CV and the ratio of the CVs of the IG distributions. Three confidence intervals for the CV of the IG distribution using the Score method, the Wald method and the percentile bootstrap confidence interval were presented by Kaewprasert et al. (2020). Later, Kaewprasert et al. (2023) proposed four confidence intervals for the ratio of the CVs of the IG distributions using the percentile bootstrap, fiducial quantities (FQs), and the Bayesian methods by the Jeffreys and uniform priors. We can applied these confidence intervals for the ratio of the CVs to test the statistical hypothesis for the ratio of the CVs.

The main objective of this paper is to propose the statistical methods for testing the ratio of the CVs for the IG distributions by using the interval estimation for the ratio of the CVs. Three confidence intervals for the ratio of the CVs are considered in order to test the ratio of the CVs: the FQ confidence interval and the Bayesian methods by the Jeffreys and uniform priors. The performance of these statistical methods was conducted by simulation study. By considering the simulation results, statistical methods with high power of a test that attained a nominal significance level are recommended for user.

The structure of this paper is as follows. The point estimation of parameters in an IG distribution are reviewed in the second section. In the third section, we present the methods for testing the ratio of the CVs of the IG distributions. The simulation study and results are discussed in the fourth section. The fifth section shows the application of the proposed statistical tests to the annual rainfall amounts in Buriram and Chaiyaphum, Thailand. Discussion and conclusions are presented in the final section.

Point Estimation of Parameters in an Inverse Gamma Distribution

The point estimation for parameters of an IG distribution is explained in this section. Let $X_1,\ldots,X_n$ be a random sample from the IG distribution with the shape parameter σ and scale parameter β The probability density function of X is given by

$$f(x;\alpha,\beta)=\frac{\beta^{\alpha}}{\Gamma(\alpha)}x^{-\alpha-1}\exp\left(-\frac{\beta}{x}\right),\quad x>0,\ \alpha>0,\ \beta>0.$$

The population mean $E(X)\,\beta = /(\alpha-1)$ for $\alpha>1$ population variance $Var(X)=\beta^2/\left[(\alpha-1)^2(\alpha-2)\right]$ for $\alpha>2$ and the CV, $\tau = 1/\sqrt{\alpha-2}.$

Suppose that $X_{ij}=(X_{i1},\ldots,X_{in});\ i=1,2,\ j=1,2,\ldots n_i$ is a vector of random samples from an IG distribution denoted as $X_{ij}=IG\ (\sigma_i,\beta_i)$ The CV of X_{ij} is $\tau_i = 1/\sqrt{\alpha_i-2},$ and the ratio of the CVs and X_{ij} are independent, which can be shown as follows:

$$\theta=\frac{\tau_1}{\tau_2}=\frac{\sqrt{\alpha_2-2}}{\sqrt{\alpha_1-2}}. \tag{1}$$

The log-likelihood function can be expressed as

$$\ln L(\alpha_i,\beta_i)=-\sum_{j=1}^{n_i}\frac{\beta_i}{X_{ij}}-(\alpha_i+1)\sum_{j=1}^{n_i}\ln X_{ij}-n_i\ln\Gamma(\alpha_i)+n_i\alpha_i\ln\beta_i.$$

The maximum likelihood estimators of α_i and β_i are given by

$$\hat{\alpha}_i = \psi^{-1}\left(\ln n\alpha_{i0} - \ln \sum_{j=1}^{n_i} X_{ij}^{-1} - \frac{\sum_{j=1}^{n_i} \ln X_{ij}}{n_i} \right) \qquad (2)$$

where $\psi(\cdot)$ is the digamma distribution, and $\alpha_{i0} = \frac{u_i^2}{v_i} + 2$ based on the moment of method estimation proposed by Llera and Beckmann (2016) for the shape parameter to initialize α_{i0} in (2); where $v_i = \frac{1}{n_i}\sum_{j=1}^{n_i}(x_{ij} - u_i)^2$ and $u_i = \frac{1}{n_i}\sum_{j=1}^{n_i} x_{ij}$ are the variance and meancalculated from the observed data $x_{ij} = (x_{i1,} x_{i2,...,} x_{in_i})$, and

$$\hat{\beta}_i = \frac{n_i \hat{\alpha}_i}{\sum_{j=1}^{n_i} X_{ij}^{-1}},$$

respectively.

Statistical Methods for Testing the Coefficient of Variation of the Inverse Gamma Distribution

Let $X_1,...,X_n$ be an independent and identically distributed random sample of size n from the IG distribution with the shape parameter α and scale parameter β. We want to test the ratio of CVs. The null and alternative hypotheses are defined as follows:

$$X_0 : \theta = \theta_0 \text{ versus } X_1 : \theta \neq \theta_0,$$

where $\theta = \tau_1/\tau_2$ and θ_0 is the hypothesized value of the ratio of CVs. In this section, we discuss three test statistics for the ratio of CVs based on the FQ confidence interval and Bayesian method.

The FQ Confidence Interval

Krishnamoorthy and Wang (2016) obtained a gamma distribution based on cube root-transformed samples and approximated fiducial quantities (FQs). The cube root-transformed samples are approximately normally distributed. Let G: *Gamma* (α_i, β_i) with shape parameter α_i and scale parameter $1/\beta_i$. Therefore, $X_i = 1/G_i$ is an IG distribution.

Suppose that $Y_i = G_i^{1/3}$; $i = 1, 2$, then Y_i is an approximately normal distribution (Wilson and Hilferty, 1931). Thus, it is transformed to an IG distribution accordingly. From $Y_i = G_i^{1/3}$ and $X_i = 1/G_i$, then $Y_i = (1/G_i)^{-1/3} = X_i^{-1/3}$ is approximately normal distribution with mean μ_i and variance σ_i^2, denoted as Y_i ; $N(\mu_i, \sigma_i^2)$ μ_i and σ_i^2 are respectively expressed as σ_i and β_i (Krishnamoorthy and Wang, 2006; Wilson and Hilferty, 1931):

$$\mu_i = \left(\frac{\alpha_i}{\beta_i}\right)^{1/3}\left(1 - \frac{1}{9\alpha_i}\right) \tag{3}$$

and

$$\sigma_i^2 = \frac{1}{9\alpha_i^{1/3}\beta_i^{2/3}}.$$

Define $\bar{Y}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} Y_{ij}$ and $S_i^2 = \frac{1}{n_i - 1}\sum_{j=1}^{n_i}(Y_{ij} - \bar{Y})^2$. Therefore, the sample mean and variance of Y_i are shown as follows:

$$\bar{Y}_i \overset{d}{=} \mu_i + Z_i \frac{\sigma_i}{\sqrt{n_i}},$$

and

$$S_i^2 \overset{d}{=} \sigma_i^2 \frac{\chi^2_{n_i - 1}}{(n_i - 1)},$$

where Z_i and χ^2_{ni-1} are the standard normal and chi-square distributions, respectively. The FQs of μ_i and σ_i^2 are respectively derived as follows:

$$F_{\mu_i} = \bar{y}_i + \frac{Z_i\sqrt{n_i - 1}}{\sqrt{\chi^2_{n_i-1}}}\frac{s_i}{\sqrt{n_i}}$$

and

$$F_{\sigma_i^2} = \frac{(n_i - 1)s_i^2}{\chi^2_{n_i - 1}},$$

where $\bar{y}_i$ and s_i^2 denote the observed values of $\bar{Y}_i$ and S_i^2 respectively. The set of Equation (3) was solved for α_i and β_i Then, we obtain

$$\alpha_i = \frac{1}{9}\left[\left(1 + \frac{\mu_i^2}{2\sigma_i^2}\right) + \left(\left(1 + \frac{\mu_i^2}{2\sigma_i^2}\right)^2 - 1\right)^{1/2}\right]$$

and

$$\beta_i = \frac{1}{27\alpha_i^{1/2}\left(\sigma_i^2\right)^{3/2}}, \tag{4}$$

respectively. Hence, the FQs for α_i are be derived as follows: (Krishnamoorthy and Wang, 2016)

$$F_{\alpha_i} = \frac{1}{9}\left[\left(1+\frac{F_{\mu_i}^2}{2F_{\sigma_i^2}}\right)+\left(\left(1+\frac{F_{\mu_i}^2}{2F_{\sigma_i^2}}\right)^2-1\right)^{1/2}\right].$$

Subsequently, the FQs for θ is

$$F_\theta = \frac{\sqrt{F_{\alpha_2}-2}}{\sqrt{F_{\alpha_1}-2}}.$$

Let γ be the significance level. Therefore, the $(1-\gamma)$ 100% confidence interval for θ by the FQs is given by

$$CI_F = [L_F, U_F] = [F_\theta(\gamma/2), F_\theta(1-(\gamma/2))], \tag{5}$$

where $F_\theta(\gamma/2)$ and $F_\theta(1-(\gamma/2))$ are the $100(\gamma/2)^{\text{th}}$ and $100(1-(\gamma/2))^{\text{th}}$ percentiles of the distribution of F_θ respectively. The algorithm for constructing the FQ confidence interval is as follows:

Algorithm 1

Step 1. Generate x_{ij} from $IG(\alpha_i, \beta_i), i = 1,2, j = 1,\ldots,n_i$
Step 2. Calculate $y_{ij} = x_{ij}^{-1/3}$
Step 3. Generate Z_i and $\chi^2_{n_i-1}$
Step 4. Compute F_{μ_i}, $F_{\alpha_i^2}$, F_{α_i} and F_θ
Step 5. Repeat Steps 3-4. 5,000 times
Step 6. Compute $(1-\gamma)100\%$ confidence interval for θ by using Equation (5).

Therefore, the null hypothesis, $H_0: \theta = \theta_0$ will be rejected if

$$\theta_0 < F_\theta(\gamma/2) \text{ or } \theta_0 > F_\theta(1-\gamma/2).$$

The Bayesian methods

We consider a Bayesian posterior density function

$$\pi(\theta/y_i) \propto L(\theta, y_i)\,\pi(\theta),$$

where $L(\theta, y_i)$ is the likelihood function and $\pi(\theta)$ is the prior. Assume that $Y_i = X_i^{-1/3}$ has a normal distribution, then the likelihood function of Y_i is defined by

$$L(\mu_i,\sigma_i^2)\propto\left(\sigma_i^2\right)^{-n_i/2}\exp\left(-\frac{1}{2\sigma_i^2}\sum_{j=1}^{n_i}(y_{ij}-\mu_i)^2\right).$$

Therefore, the Bayesian method by the Jeffreys and uniform priors are accordingly applied to construct the confidence intervals for θ.

1) The Jeffreys prior

This prior is defined by $\pi(\theta)=\sqrt{\det(I(\theta))}$, where $I(\theta)$ is the Fisher information matrix (Jeffreys, 1961). Therefore, the Fisher information matrix is expressed as

$$I(\mu_i,\sigma_i^2)=\begin{bmatrix} n_i/\sigma_i^2 & 0 \\ 0 & n_i/2\sigma_i^2 \end{bmatrix}$$

Consider $X_i^{-1/3}=Y_i=\left(Y_1=X_1^{-1/3},\dots,Y_{n_i}=X_{n_i}^{-1/3}\right)$, modeled as Y_i ; $N(\mu_i,\ \sigma_i^2)$ where σ_i^2 is assumed known. The Fisher information of μ_i is defined by $I(\mu_i)=1/\sigma_i^2$. Then, the Jeffreys prior of μ_i is $\pi(\mu_i\,|\,\sigma_i^2)\propto\sqrt{1/\sigma_i^2}\propto$ const. In the same way, the Jeffreys prior of σ_i^2 is $\pi(\sigma_i^2)\propto 1/\sigma_i^2$ Therefore, the Jeffreys prior is obtained by

$$\pi(\mu_i,\sigma_i^2)\propto\text{const}\times 1/\sigma_{\text{i}}^2\propto\sigma_i^{-2}$$

which is combined with the likelihood function, the posterior density function is given by

$$\pi(\mu_i,\sigma_i^2\,|\,y_i)\propto\sigma_i^{-2}\left(\sigma_i^2\right)^{-n_i/2}\exp\left(-\frac{1}{2\sigma_i^2}\sum_{j=1}^{n_i}(y_{ij}-\mu_i)^2\right).$$

The irrespective marginal posteriors are normal and IG distributions because μ_i and σ_i^2 are independent. Dongchu and Keying (1996) defined them as follows:

$$\pi(\mu_i\,|\,\sigma_i^2,y_i)_J\ ;\ N(\hat{\mu}_i,\sigma_i^2/n_i) \tag{6}$$

and

$$\pi(\sigma_i^2\,|\,y_i)_J\ ;\ IG(n_i/2,y_{n_i}/2), \tag{7}$$

where $\hat{\mu}_i=\frac{1}{n_i}\sum_{j=1}^{n_i}y_{ij}$ and $y_{n_i}=\sum_{j=1}^{n_i}(y_{ij}-\bar{y}_{ij})^2$.

To conduct the Bayesian method by the Jeffreys prior, α_1 and α_2 are substituted in Equation (4) by $(\mu_i\,|\sigma_i^2,y_i)_J$ and $(\sigma_i^2|y_i)_J$ defined in Equations (6) and (7), respectively.

$$(\alpha_i)_J = \frac{1}{9}\left[\left(1+\frac{(\mu_i \mid \sigma_i^2, y_i)_J^2}{2(\sigma_i^2 \mid y_i)_J}\right)+\left(\left(1+\frac{(\mu_i \mid \sigma_i^2, y_i)_J^2}{2(\sigma_i^2 \mid y_i)_J}\right)^2-1\right)^{1/2}\right], \text{ for } i=1,2$$

and we compute θ_J by $(\alpha_i)_J$ based on Equation (1). Then,

$$\theta_J = \frac{\sqrt{(\alpha_2)_J - 2}}{\sqrt{(\alpha_1)_J - 2}}.$$

Therefore, $(1-\gamma)100\%$ confidence interval for θ based on the Bayesian method by the Jeffreys prior is given by

$$CI_J = \left[\theta_J(\gamma/2), \theta_J(1-(\gamma/2))\right], \tag{8}$$

where $\theta_J(\gamma/2)$ and $\theta_J(1-(\gamma/2))$ are the $100(\gamma/2)^{\text{th}}$ and $100-(\gamma/2)^{\text{th}}$ percentiles of the distribution of θ_J, respectively. The algorithm for constructing the Bayesian confidence interval by the Jeffreys prior is as follows:

Algorithm 2

Step 1. Generate x_{ij} from $IG(\alpha_i, \beta_i), i = 1,2, j = 1,\ldots,n_i$
Step 2. Calculate $y_{ij} = x_{ij}^{-1/3}$
Step 3. Computet $(\mu_i \mid \sigma_i^2, y_i)_J$ by using Equation (6)
Step 4. Compute $(\sigma_i^2, \mid y_i)_J$ by using Equation (7)
Step 5. Compute $(\alpha_i)_J$
Step 6. Compute θ_J by using $(\alpha_i)_J$ from Step 5.
Step 7. Repeat Steps 3-6. 5,000 times
Step 8. Compute $(1-\gamma)100\%$ confidence interval for θ by using Equation (8).

Therefore, the null hypothesis, $H_0 : \theta = \theta_0$ will be rejected if

$$\theta_0 < \theta_J(\gamma/2) \text{ or } \theta_0 < \theta_J(1-(\gamma/2)).$$

2) The uniform prior

The uniform priors of μ_i and σ_i^2 are $\pi(\mu_i) \propto 1$, and $\pi(\sigma_i^2) \propto 1$, respectively. Thus, the IG distribution for the Bayesian method based on uniform prior is $\pi(\mu_i, \sigma_i^2) \propto 1$, The respective marginal posteriors of μ_i and σ_i^2 are defined as (Yang and Berger, 1998)

$$\pi(\mu_i \mid \sigma_i^2, y_i)_U \; ; \; N(\hat{\mu}_i, \sigma_i^2 / n_i) \tag{9}$$

and
$$\pi(\sigma_i^2 \mid y_i)_U \; ; \; IG((n_i - 2)/2, y_{n_i}/2), \tag{10}$$

where and $\hat{\mu}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} y_{ij}$ and $y_{n_i} = \sum_{j=1}^{n_i}(y_{ij} - \bar{y}_{ij})^2$.

Then, the Bayesian confidence interval is proposed by using $(\mu_i \mid \sigma_i^2, y_i)_U$ and $(\sigma_i^2, |y_i)_U$ from Equations (9) and (10), respectively.

$$(\alpha_i)_U = \frac{1}{9}\left[\left(1 + \frac{(\mu_i, \sigma_i^2 \mid y_i)_U^2}{2(\sigma_i^2 \mid y_i)_U}\right) + \left(\left(1 + \frac{(\mu_i, \sigma_i^2 \mid y_i)_U^2}{2(\sigma_i^2 \mid y_i)_U}\right)^2 - 1\right)^{1/2}\right], \text{ for } i = 1,2$$

and the θ_U is calculated by using $(\alpha_i)_U$ based on Equation (1). Then,

$$\theta_U = \frac{\sqrt{(\alpha_2)_U - 2}}{\sqrt{(\alpha_1)_U - 2}}.$$

Therefore, $(1-\gamma)100\%$ confidence interval for θ based on the Bayesian method by the uniform prior is given by

$$CI_U = \left[\theta_U(\gamma/2), \theta_U(1-(\gamma/2))\right], \tag{11}$$

where $\theta_U(\gamma/2)$ and $\theta_U(1-(\gamma/2))$ are the $100(\gamma/2)^{th}$ and $100(1-(\gamma/2))^{th}$ percentiles of the distribution of θ_U respectively. The following algorithm is used to construct the Bayesian confidence interval by the uniform prior:

Algorithm 3

Step 1. Generate x_{ij} from $IG(\alpha_i, \beta_i), i = 1,2, j = 1,...,n_i$
Step 2. Calculate $y_{ij} = x_{ij}^{-1/3}$
Step 3. Compute $(\mu_i \mid \sigma_i^2, y_i)_U$ by using Equation (9)
Step 4. Compute $(\sigma_i^2 \mid y_i)_U$ by using Equation (10)
Step 5. Compute $(\alpha_i)_U$
Step 6. Compute θ_U by using $(\alpha_i)_U$ from Step 5.
Step 7. Repeat Steps 3-6. 5,000 times
Step 8. Compute $(1-\gamma)100\%$ confidence interval for θ by using Equation (10).

Therefore, the null hypothesis, $H_0 : \theta = \theta_0$ will be rejected if

$$\theta_0 < \theta_U(\gamma/2) \text{ or } \theta_0 < \theta_U(1-(\gamma/2)).$$

Simulation Study and Results

In this study, three statistical methods for testing the ratio of the CVs in the IG distributions are considered. Since a theoretical comparison is not possible, a Monte Carlo simulation was proceeded using the R version 4.1.3 statistical software (Ihaka and Gentleman, 1996) to compare the performance of the test statistics. These statistical methods were evaluated in terms of their attainment of empirical type I error rates and the powers of a test of their performance. The simulation results are presented only for the significant level γ = 0.05, since i) γ = 0.05 is widely applied to compare the empirical type I error rate and the power of a test ii) similar conclusions were derived for other values of γ.

Equal sample sizes were set as (n_1, n_2) = (25,25), (50,50), (75,75) and (100,100) and unequal sample sizes as (n_1, n_2) = (25,50), (50,75) and (75,100). The number of simulations was fixed at 10,000. The data were generated from two independent IG distributions with $\beta_1 = \beta_2 = 1$ and α_1 and α_2 was adjusted to obtain the required CVs, (τ_1, τ_2) We set (τ_1, τ_2) = (0.05,0.25), (0.10,0.25), (0.05,0.10), (0.15,0.25), (0.20,0.25) and (0.25,0.25). Therefore, the hypothesized values of ratio of the CVs for the IG distributions, θ_0 = 0.2, 0.4, 0.5, 0.6, 0.8 and 1.0. In case of the powers of the tests, we set the values of the ratio of CVs, $\theta = \theta_0 \pm c \text{ x } 0.05$ where $c = 0, \pm 1, \pm 2, \pm 3$ and ± 4.

From the simulation results shown in Tables 1-6, the empirical type I error rates of all test statistics were close to the nominal significance level of 0.05 for all situations. The power of a test statistics based on the Bayesian method by the Jeffreys prior were higher than those of other test statistics for equal sample sizes. In case of unequal sample sizes and $\theta < \theta_0$, the Bayesian method by the uniform prior performed well in terms of the power of a test. On the other hand, the Bayesian method by the Jeffreys prior performed better for $\theta > \theta_0$. A general pattern can be observed; when the sample size increases, the power of the test also increases and the empirical type I error rate is close to the nominal level of significance. In addition, the power of a test increases as the value of the ratio of the CVs departs from the hypothesized value of the ratio of the CVs.

We observed that for large sample sizes, the performance of the all test statistics did not differ greatly in terms of power of a test and the attainment of the nominal significance level of the test. Nevertheless, a significant difference was noticed for small sample sizes.

Empirical Application

To illustrate the application of the three statistical methods for testing the ratio of the CVs introduced in the previous section, we used two data sets on the annual rainfall amounts (millimeter: mm.). The first and second data sets were measured from the station at Buriram and Chaiyaphum, Thailand from 1998 to 2020, respectively. The data sets were reported by the Hydrology Irrigation Centers for the Upper and Lower Northeastern Regions, the Royal Irrigation Department, Thailand (http://hydro-3.rid.go.th, http://hydro-4.rid.go.th). The descriptive statistics are shown in Table 7. The distributions of the annual rainfall amounts in both provinces are right-skewed (coefficients of skewness are positive) and they have heavy-tailed distributions (coefficients of kurtosis are positive). By considering the histogram, density plot, Box and Whisker plot, and IG quantile-quantile (Q-Q) plot shown in Figures 1 and 2, the fitted distributions for the annual rainfall amounts of both provinces are not symmetric.

Table 8 reports the Akaike information criterion (AIC) (Akaike, 1974) results to check the fitting of the distributions for the annual rainfall amounts in both provinces. The AIC is defined as $AIC = -2 \ln L + 2k$, where L is the likelihood function and k is the number of parameters. To find the best fitted distribution for the annual rainfall amounts, the AIC values for several distributions were considered. The results show that the annual rainfall amounts of Buriram and Chaiyaphum follow the IG distributions because the AIC values for this distribution were the smallest. The annual rainfall amounts in Buriram had an IG distribution with shape parameter $\hat{\alpha}_1 = 60.4835$ and scale parameter $\hat{\beta}_1 = 62{,}950.10$, while the MLE for the CV is $\hat{\tau}_1 = 0.1308$. Similarly, the annual rainfall amounts in Chaiyaphum had an IG distribution with shape parameter $\hat{\alpha}_2 = 22.1316$ and scale parameter $\hat{\beta}_2 = 23{,}026.55$, while the MLE for the CV is $\hat{\tau}_2 = 0.2229$. Therefore, the ratio of the CVs is $\hat{\theta} = 0.5868$.

Table 1: Empirical Type I Error Rates (Bold Numeric) and Powers of Tests (Not Bold Numeric) for $\theta_0 = 0.20$

(n_1, n_2)	Method	θ								
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
(25,25)	FQ	1.0000	0.9999	0.8469	0.2247	**0.0474**	0.1776	0.4645	0.7351	0.8921
	Jeffreys	1.0000	0.9999	0.8638	0.2429	**0.0532**	0.1855	0.4774	0.7463	0.8986
	Uniform	1.0000	0.9999	0.8259	0.2059	**0.0432**	0.1678	0.4518	0.7211	0.8871
(50,50)	FQ	1.0000	1.0000	0.9913	0.4413	**0.0475**	0.3111	0.7666	0.9649	0.9952
	Jeffreys	1.0000	1.0000	0.9923	0.4522	**0.0489**	0.3164	0.7700	0.9665	0.9955
	Uniform	1.0000	1.0000	0.9899	0.4296	**0.0444**	0.3058	0.7611	0.9633	0.9948
(75,75)	FQ	1.0000	1.0000	0.9999	0.6201	**0.0529**	0.4546	0.9083	0.9937	1.0000
	Jeffreys	1.0000	1.0000	0.9999	0.6286	**0.0550**	0.4569	0.9102	0.9939	1.0000
	Uniform	1.0000	1.0000	0.9997	0.6116	**0.0516**	0.4517	0.9069	0.9937	1.0000
(100,100)	FQ	1.0000	1.0000	1.0000	0.7524	**0.0475**	0.5616	0.9691	0.9998	1.0000
	Jeffreys	1.0000	1.0000	1.0000	0.7580	**0.0490**	0.5625	0.9703	0.9997	1.0000
	Uniform	1.0000	1.0000	1.0000	0.7468	**0.0470**	0.5569	0.9698	0.9997	1.0000
(25,50)	FQ	1.0000	1.0000	0.9515	0.3434	**0.0496**	0.2049	0.5692	0.8484	0.9618
	Jeffreys	1.0000	1.0000	0.9504	0.3380	**0.0535**	0.2279	0.6077	0.8682	0.9694
	Uniform	1.0000	1.0000	0.9529	0.3511	**0.0462**	0.1790	0.5273	0.8226	0.9500
(50,75)	FQ	1.0000	1.0000	0.9978	0.5521	**0.0479**	0.3584	0.8292	0.9838	0.9990
	Jeffreys	1.0000	1.0000	0.9976	0.5513	**0.0503**	0.3729	0.8409	0.9853	0.9992
	Uniform	1.0000	1.0000	0.9977	0.5523	**0.0467**	0.3427	0.8188	0.9814	0.9990
(75,100)	FQ	1.0000	1.0000	0.9999	0.6998	**0.0477**	0.4932	0.9450	0.9983	1.0000
	Jeffreys	1.0000	1.0000	0.9999	0.6991	**0.0492**	0.5021	0.9476	0.9987	1.0000
	Uniform	1.0000	1.0000	0.9999	0.7002	**0.0472**	0.4840	0.9417	0.9983	1.0000

Table 2: Empirical Type I Error Rates (Bold Numeric) and Powers of Tests (Not Bold Numeric) for $\theta_0 = 0.40$

(n_1, n_2)	Method	θ								
		0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
(25,25)	FQ	0.8441	0.5207	0.2333	0.0864	**0.0500**	0.0830	0.1753	0.3072	0.4531
	Jeffreys	0.8610	0.5435	0.2494	0.0949	**0.0541**	0.0891	0.1844	0.3207	0.4673
	Uniform	0.8283	0.4958	0.2137	0.0756	**0.0446**	0.0768	0.1653	0.2951	0.4385
(50,50)	FQ	0.9910	0.8386	0.4466	0.1291	**0.0511**	0.1235	0.3077	0.5433	0.7643
	Jeffreys	0.9919	0.8456	0.4581	0.1351	**0.0532**	0.1263	0.3151	0.5489	0.7685
	Uniform	0.9897	0.8288	0.4357	0.1240	**0.0483**	0.1199	0.3038	0.5381	0.7600
(75,75)	FQ	0.9996	0.9556	0.6201	0.1846	**0.0479**	0.1545	0.4350	0.7316	0.9088
	Jeffreys	0.9997	0.9588	0.6289	0.1899	**0.0501**	0.1574	0.4387	0.7363	0.9090
	Uniform	0.9995	0.9535	0.6102	0.1797	**0.0458**	0.1512	0.4312	0.7285	0.9069
(100,100)	FQ	0.9999	0.9877	0.7453	0.2323	**0.0482**	0.1907	0.5484	0.8458	0.9664
	Jeffreys	0.9999	0.9882	0.7501	0.2368	**0.0508**	0.1909	0.5497	0.8477	0.9673
	Uniform	0.9999	0.9869	0.7397	0.2273	**0.0465**	0.1880	0.5442	0.8445	0.9668
(25,50)	FQ	0.9513	0.7047	0.3454	0.1142	**0.0499**	0.0874	0.2039	0.3769	0.5421
	Jeffreys	0.9495	0.7003	0.3390	0.1123	**0.0520**	0.1001	0.2284	0.4164	0.5788
	Uniform	0.9534	0.7136	0.3530	0.1180	**0.0479**	0.0753	0.1770	0.3405	0.5019
(50,75)	FQ	0.9985	0.9166	0.5319	0.1653	**0.0483**	0.1367	0.3498	0.6246	0.8306
	Jeffreys	0.9986	0.9154	0.5334	0.1657	**0.0496**	0.1449	0.3635	0.6416	0.8401
	Uniform	0.9986	0.9162	0.5343	0.1651	**0.0456**	0.1295	0.3355	0.6104	0.8196
(75,100)	FQ	0.9999	0.9788	0.6938	0.2073	**0.0487**	0.1762	0.4793	0.7789	0.9468
	Jeffreys	0.9999	0.9788	0.6931	0.2075	**0.0493**	0.1812	0.4901	0.7859	0.9490
	Uniform	0.9998	0.9784	0.6918	0.2082	**0.0474**	0.1688	0.4702	0.7719	0.9413

Table 3: Empirical Tpe I Error Rates (Bold Numeric) and Powers of Tests (Not Bold Numeric) for $\theta_0 = 0.50$

(n_1, n_2)	Method	θ								
		0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70
(25,25)	FQ	0.6746	0.3902	0.1804	0.0757	**0.0497**	0.0721	0.1397	0.2414	0.3658
	Jeffreys	0.6903	0.4057	0.1927	0.0828	**0.0533**	0.0792	0.1495	0.2529	0.3812
	Uniform	0.6568	0.3714	0.1689	0.0690	**0.0437**	0.0667	0.1308	0.2288	0.3508
(50,50)	FQ	0.9359	0.6846	0.3227	0.1097	**0.0454**	0.0945	0.2381	0.4361	0.6357
	Jeffreys	0.9380	0.6926	0.3300	0.1121	**0.0485**	0.0982	0.2429	0.4432	0.6442
	Uniform	0.9322	0.6760	0.3164	0.1043	**0.0439**	0.0906	0.2308	0.4286	0.6267
(75,75)	FQ	0.9903	0.8544	0.4745	0.1431	**0.0494**	0.1273	0.3395	0.5997	0.8161
	Jeffreys	0.9911	0.8572	0.4804	0.1474	**0.0504**	0.1304	0.3465	0.6055	0.8191
	Uniform	0.9903	0.8501	0.4667	0.1405	**0.0472**	0.1260	0.3356	0.5928	0.8141
(100,100)	FQ	0.9987	0.9385	0.5796	0.1735	**0.0474**	0.1591	0.4370	0.7311	0.9099
	Jeffreys	0.9987	0.9402	0.5807	0.1770	**0.0495**	0.1593	0.4417	0.7334	0.9111
	Uniform	0.9987	0.9374	0.5740	0.1710	**0.0470**	0.1566	0.4315	0.7268	0.9075
(25,50)	FQ	0.8214	0.5258	0.2471	0.0922	**0.0521**	0.0769	0.1532	0.2728	0.4323
	Jeffreys	0.8137	0.5123	0.2386	0.0893	**0.0553**	0.0894	0.1784	0.3067	0.4735
	Uniform	0.8296	0.5391	0.2571	0.0959	**0.0497**	0.0645	0.1327	0.2382	0.3869
(50,75)	FQ	0.9716	0.7664	0.4061	0.1305	**0.0531**	0.1101	0.2725	0.4964	0.7035
	Jeffreys	0.9717	0.7635	0.4025	0.1297	**0.0556**	0.1181	0.2882	0.5133	0.7193
	Uniform	0.9721	0.7697	0.4099	0.1341	**0.0507**	0.1024	0.2596	0.4799	0.6878
(75,100)	FQ	0.9954	0.9015	0.5255	0.1593	**0.0489**	0.1352	0.3695	0.6526	0.8534
	Jeffreys	0.9955	0.9008	0.5250	0.1605	**0.0499**	0.1416	0.3810	0.6634	0.8574
	Uniform	0.9955	0.9022	0.5268	0.1630	**0.0482**	0.1290	0.3588	0.6418	0.8463

Table 4: Empirical Type I Error Rates (Bold Numeric) and Powers of Tests (Not Bold Numeric) for $\theta_0 = 0.60$

(n_1, n_2)	Method	θ								
		0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80
(25,25)	FQ	0.4089	0.2282	0.1119	0.0666	**0.0485**	0.0617	0.1088	0.1659	0.2526
	Jeffreys	0.4310	0.2441	0.1220	0.0724	**0.0535**	0.0680	0.1158	0.1769	0.2636
	Uniform	0.3836	0.2105	0.1037	0.0595	**0.0446**	0.0567	0.1007	0.1562	0.2373
(50,50)	FQ	0.7175	0.4353	0.2036	0.0840	**0.0472**	0.0825	0.1678	0.2910	0.4618
	Jeffreys	0.7279	0.4440	0.2104	0.0878	**0.0495**	0.0853	0.1723	0.2981	0.4700
	Uniform	0.7066	0.4224	0.1934	0.0791	**0.0464**	0.0799	0.1630	0.2863	0.4540
(75,75)	FQ	0.8816	0.6074	0.2911	0.1021	**0.0478**	0.0953	0.2307	0.4213	0.6333
	Jeffreys	0.8855	0.6160	0.2972	0.1050	**0.0500**	0.0973	0.2341	0.4274	0.6360
	Uniform	0.8757	0.5984	0.2858	0.0980	**0.0475**	0.0933	0.2264	0.4178	0.6288
(100,100)	FQ	0.9552	0.7386	0.3746	0.1208	**0.0507**	0.1185	0.2925	0.5390	0.7542
	Jeffreys	0.9568	0.7428	0.3795	0.1254	**0.0518**	0.1191	0.2945	0.5433	0.7543
	Uniform	0.9538	0.7325	0.3668	0.1191	**0.0504**	0.1163	0.2890	0.5368	0.7493
(25,50)	FQ	0.5863	0.3402	0.1699	0.0766	**0.0485**	0.0650	0.1153	0.2028	0.2897
	Jeffreys	0.5773	0.3329	0.1631	0.0773	**0.0517**	0.0767	0.1348	0.2262	0.3190
	Uniform	0.5937	0.3502	0.1748	0.0791	**0.0458**	0.0567	0.0998	0.1764	0.2576
(50,75)	FQ	0.8199	0.5310	0.2540	0.0945	**0.0494**	0.0782	0.1977	0.3443	0.5189
	Jeffreys	0.8194	0.5281	0.2525	0.0943	**0.0517**	0.0850	0.2084	0.3602	0.5375
	Uniform	0.8197	0.5323	0.2544	0.0950	**0.0484**	0.0736	0.1849	0.3258	0.5022
(75,100)	FQ	0.9312	0.6901	0.3434	0.1134	**0.0500**	0.1024	0.2568	0.4832	0.6892
	Jeffreys	0.9300	0.6920	0.3458	0.1135	**0.0506**	0.1067	0.2635	0.4917	0.6984
	Uniform	0.9306	0.6913	0.3433	0.1135	**0.0485**	0.0994	0.2477	0.4722	0.6804

Table 5: Empirical Type I Error Rates (Bold Numeric) and Powers of Tests (Not Bold Numeric) for θ_0 = 0.80

(n_1, n_2)	Method	θ								
		0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
(25,25)	FQ	0.2326	0.1398	0.0855	0.0569	**0.0488**	0.0565	0.0788	0.1141	0.1646
	Jeffreys	0.2466	0.1501	0.0939	0.0609	**0.0536**	0.0620	0.0854	0.1234	0.1759
	Uniform	0.2148	0.1278	0.0798	0.0523	**0.0449**	0.0503	0.0716	0.1062	0.1546
(50,50)	FQ	0.4275	0.2444	0.1300	0.0637	**0.0526**	0.0627	0.1171	0.1991	0.2885
	Jeffreys	0.4382	0.2541	0.1355	0.0681	**0.0539**	0.0653	0.1216	0.2044	0.2954
	Uniform	0.4160	0.2371	0.1258	0.0623	**0.0490**	0.0589	0.1127	0.1925	0.2803
(75,75)	FQ	0.5969	0.3555	0.1731	0.0745	**0.0496**	0.0783	0.1480	0.2684	0.4207
	Jeffreys	0.6047	0.3626	0.1754	0.0779	**0.0519**	0.0795	0.1517	0.2717	0.4233
	Uniform	0.5906	0.3495	0.1686	0.0723	**0.0482**	0.0767	0.1459	0.2633	0.4142
(100,100)	FQ	0.7369	0.4614	0.2239	0.0844	**0.0521**	0.0899	0.1854	0.3456	0.5323
	Jeffreys	0.7416	0.4694	0.2285	0.0860	**0.0532**	0.0898	0.1871	0.3482	0.5369
	Uniform	0.7311	0.4568	0.2222	0.0812	**0.0509**	0.0886	0.1826	0.3421	0.5284
(25,50)	FQ	0.3402	0.1991	0.1176	0.0668	**0.0517**	0.0615	0.0798	0.1291	0.1895
	Jeffreys	0.3306	0.1942	0.1146	0.0650	**0.0552**	0.0715	0.0953	0.1468	0.2185
	Uniform	0.3499	0.2062	0.1217	0.0661	**0.0488**	0.0527	0.0689	0.1124	0.1639
(50,75)	FQ	0.5352	0.3219	0.1549	0.0758	**0.0477**	0.0695	0.1265	0.2175	0.3327
	Jeffreys	0.5328	0.3185	0.1543	0.0750	**0.0498**	0.0750	0.1361	0.2317	0.3494
	Uniform	0.5355	0.3234	0.1559	0.0758	**0.0466**	0.0634	0.1168	0.2044	0.3156
(75,100)	FQ	0.6776	0.4119	0.2022	0.0867	**0.0511**	0.0779	0.1677	0.2989	0.4473
	Jeffreys	0.6768	0.4136	0.2028	0.0872	**0.0521**	0.0811	0.1728	0.3103	0.4593
	Uniform	0.6744	0.4108	0.2020	0.0869	**0.0492**	0.0763	0.1615	0.2911	0.4360

Table 6: Empirical Type I Error Rates (Bold Numeric) and Powers of Tests (Not Bold Numeric) for $\theta_0 = 1.00$

(n_1, n_2)	Method	θ								
		0.80	**0.85**	**0.90**	**0.95**	**1.00**	**1.05**	**1.10**	**1.15**	**1.20**
(25,25)	FQ	0.1522	0.1039	0.0719	0.0537	**0.0496**	0.0495	0.0691	0.0901	0.1234
	Jeffreys	0.1652	0.1126	0.0788	0.0594	**0.0547**	0.0553	0.0755	0.0972	0.1312
	Uniform	0.1421	0.0961	0.0654	0.0499	**0.0439**	0.0446	0.0632	0.0810	0.1149
(50,50)	FQ	0.2863	0.1639	0.1015	0.0594	**0.0487**	0.0593	0.0937	0.1365	0.1973
	Jeffreys	0.2959	0.1711	0.1054	0.0621	**0.0512**	0.0623	0.0980	0.1425	0.2056
	Uniform	0.2763	0.1594	0.0968	0.0566	**0.0467**	0.0568	0.0902	0.1315	0.1891
(75,75)	FQ	0.3951	0.2308	0.1278	0.0688	**0.0509**	0.0635	0.1116	0.1853	0.2777
	Jeffreys	0.3993	0.2363	0.1305	0.0704	**0.0525**	0.0654	0.1155	0.1900	0.2833
	Uniform	0.3899	0.2265	0.1251	0.0659	**0.0496**	0.0626	0.1075	0.1811	0.2709
(100,100)	FQ	0.5051	0.3018	0.1517	0.0747	**0.0472**	0.0640	0.1344	0.2331	0.3610
	Jeffreys	0.5108	0.3042	0.1560	0.0760	**0.0482**	0.0643	0.1379	0.2373	0.3640
	Uniform	0.5007	0.2984	0.1503	0.0730	**0.0458**	0.0620	0.1330	0.2297	0.3556
(25,50)	FQ	0.2138	0.1405	0.0874	0.0595	**0.0488**	0.0563	0.0717	0.0991	0.1320
	Jeffreys	0.2058	0.1346	0.0842	0.0605	**0.0523**	0.0634	0.0838	0.1148	0.1544
	Uniform	0.2215	0.1464	0.0896	0.0590	**0.0463**	0.0490	0.0605	0.0835	0.1108
(50,75)	FQ	0.3360	0.2036	0.1163	0.0673	**0.0477**	0.0627	0.0947	0.1526	0.2187
	Jeffreys	0.3352	0.2016	0.1130	0.0676	**0.0491**	0.0669	0.1022	0.1617	0.2359
	Uniform	0.3400	0.2060	0.1159	0.0683	**0.0453**	0.0583	0.0876	0.1416	0.2094
(75,100)	FQ	0.4466	0.2615	0.1383	0.0675	**0.0539**	0.0666	0.1198	0.1981	0.3103
	Jeffreys	0.4470	0.2600	0.1382	0.0677	**0.0562**	0.0707	0.1270	0.2077	0.3212
	Uniform	0.4494	0.2627	0.1394	0.0678	**0.0531**	0.0640	0.1147	0.1904	0.2996

Our interest was in testing the ratio of the CVs of the annual rainfall amounts in Buriram and Chaiyaphum. Suppose the researcher wanted to test the claim that the ratio of the CVs equals 0.6. The null and alternative hypotheses are respectively given as follows:

$$H_0: \theta = 0.5 \text{ versus } H_1: \theta \neq 0.5$$

The lower and upper critical values of both test statistics were shown in Table 9. The null hypothesis H_0 did not been rejected since $\theta_0 \in [0.4668.\ 1/1862]$. $\theta_0 \in [0.4840.\ 1.1848]$ and $\theta_0 \in [0.4618.\ 1.1944]$. using test statistics based on the FQ and Bayesian methods by the Jeffreys and uniform priors, respectively. We conclude that the ratio of the CVs of the annual rainfall amounts in Buriram and Chaiyaphum does not differ from 0.5 at the 0.05 significance level.

Table 7: Descriptive Statistics of the Annual Rainfall Amounts in Buriram and Chaiyaphum

Provinces	Sample sizes	Mean	SD.	Skewness	Kurtosis
Buriram	23	1069.40	195.49	1.473	2.521
Chaiyaphum	23	1088.44	245.79	0.886	0.946

Table 8: Results of AIC for the Annual Rainfall Amounts in Buriram and Chaiyaphum

Provinces	Normal	Cauchy	Exponential	Weibull	Gamma	Inverse Gamma
Buriram	310.9230	310.3707	368.8432	315.6505	307.7593	**305.3693**
Chaiyaphum	321.4549	328.0113	369.6550	323.7112	319.2704	**318.2490**

Table 9: Critical Values of Test Statistics Based on the FQ and Bayesian Methods by the Jeffreys and Uniform Priors for the Significance Level of 0.05

Method	Critical Values	
	Lower	Upper
FQ	0.4668	1.1862
Jeffreys	0.4840	1.1848
Uniform	0.4618	1.1944

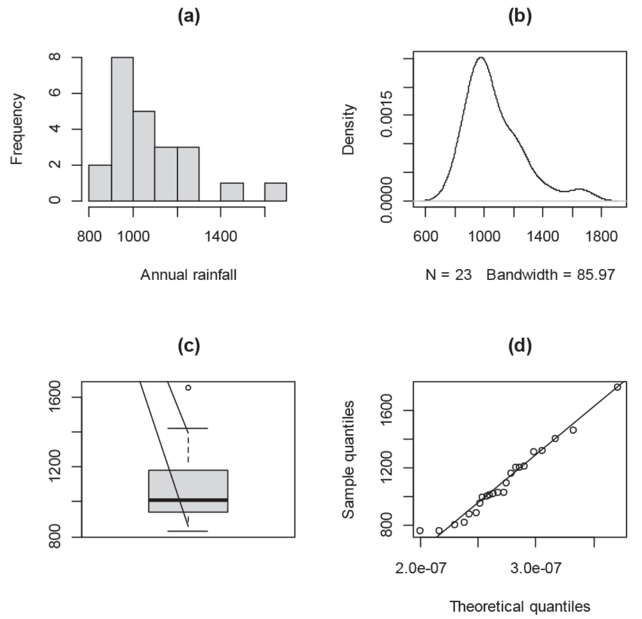

Figure 1: (a) Histogram (b) Density Plot (c) Box and Whisker Plot
(d) Inverse Gamma Q-Q Plot of the Annual Rainfall Amounts in Buriram, Thailand

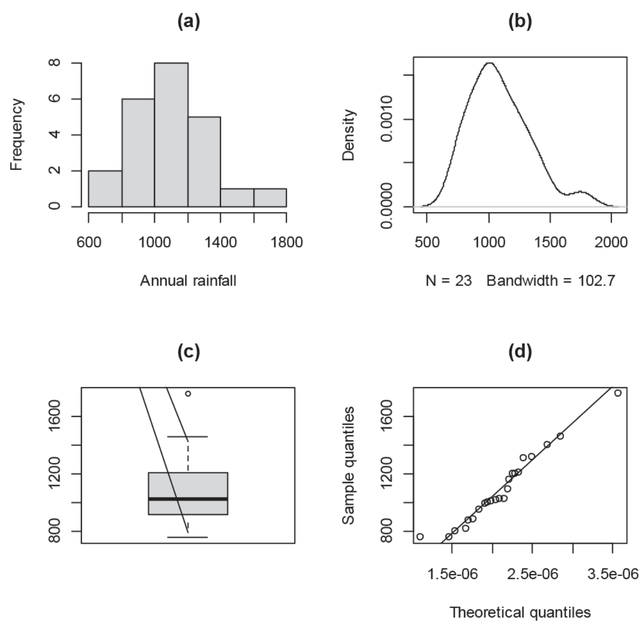

Figure 2: (a) Histogram (b) Density Plot (c) Box and Whisker Plot
(d) Inverse Gamma Q-Q Plot of the Annual Rainfall Amounts in Chaiyaphum, Thailand

Discussion and Conclusions

The goal of this study is to recognize potential statistical methods that can be recommended to researchers or users for testing the ratio of the CVs of the IG distributions. A general pattern was noticed; as the sample size increased, the power of a test also increased and the empirical type I error rates was close to the significance level of 0.05. In addition, the power increased as the value of the ratio of the CVs departed from the hypothesized value of the ratio of the CVs. Furthermore, the researchers can applied the proposed methods for testing the ratio of the CVs in the IG distributions with other data sets fitted well to the IG distributions. For example, the IG distribution has been used for the hitting time distribution of a Wiener process. For the future research, it is interested in the one-tailed hypothesis testing.

The previous research studied by Kaewprasert et al. (2023) found that the confidence intervals for the ratio of the CVs of the IG distributions constructed with the Bayesian method based on the uniform prior and fiducial quantities performed better than those constructed with the Bayesian method based on the Jeffreys prior and percentile bootstrap method. The simulation results from this paper which studied the testing the ratio of the CVs is different from the previous work related the confidence intervals.

In this study, three statistical methods for testing the ratio of the CVs of the IG distributions were derived. Based on the simulation results, it is evident that the test statistics based on Bayesian method by the Jeffreys prior performed well in terms of the empirical type I error rate and power of a test for equal sample sizes. In case of unequal sample sizes, the test statistics based on Bayesian method by the Jeffreys and uniform priors performed well in term of the powers of the tests for $\theta < \theta_0$ and $\theta > \theta_0$ respectively. The annual rainfall amounts from the northeastern of Thailand were applied to illustrate the efficacies of the proposed statistical methods.

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