Early Algebraic Reasoning: Developing Relational Thinking Through The Flow of Lesson

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ABSTRACT

Almost students in high school level face many difficulties while learning Algebra. Nowadays, educators try to solve this problem through integrating algebraic reasoning to the elementary school curriculum. Relational thinking is conceived as the main branch of early algebraic reasoning. Hence, this work aimed to identify the relational thinking of students in grade one. This relational thinking developed through Flow of Lesson (connection between the real world of students and the real world of mathematics) in the classroom of mathematics implementing Lesson Study and Open Approach. Qualitative approach that uses a participatory ethnography study as a member of Lesson Study team. This Lesson Study team worked collaboratively used Japanese mathematics textbooks in the Thai version. Data was collected from students' written works, audio-video tape recordings, and strengthened by field notes from the Lesson Study team. The results show: 1) real-world representation help students to underlay the relationship between numbers; 2) diagram as semi-concrete aids help students to structure number sentences; 3) mathematical representation building from real-world representation and semi-concrete aids, help students to structure number sentences in general case. Therefore, this mathematics classroom model can be one alternative solution to encourage students' relational thinking.

Keywords: Early Algebra, Flow of Lesson, Relational Think

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Introduction

Algebra is one of the common content standards that all grade schools should have [1]. Strengthening the claim, [2] stated that "the core reform of the mathematics curriculum is algebraic reform." Algebra has a role as a foundation for other mathematical content that has served as a basic knowledge prior to advanced mathematics [3,4]. It is also known as a problem-solving tool, provides a means of describing and analyzing relationships, and provides a key to characterizing and understanding the structures of mathematics [5].

Nonetheless, according to [6] high school students experience difficulties in understanding basic algebraic concepts because of postponement of learning algebra to secondary school. "Arithmetic-then-algebra" approach leaves a little cognitive room for students to negotiate an abrupt transition from years of computational work in primary and secondary schools to abstract concepts of formal high school algebra [7]. Because of that, [8] argued that students were able to begin to think algebraically much earlier than previously thought and in ways that could potentially correct the difficulties historically faced by high school algebra students. This idea related to the concept of early algebra perspective lies on Vygotsky's idea that instruction precedes cognitive develop [9]. However, this perspective did not include teaching children in elementary and middle grades the formal procedural algebra of traditional ninth grade algebra courses [7].

Mathematics educational researchers have worked together to find some ways to integrate algebraic reasoning into the K-12 grade curricula and have also argued that algebra is not limited to content, and some features do not focus on content, but students can still learn algebra through arithmetic content boundary [8,10–13]. The main objective, however, is to conceptualize algebra more broadly so that there is a shift in emphasis from the rules of learning for symbol manipulation to the development of algebraic reasoning [14]. Algebraic reasoning is concerned with the nature of algebraic thinking that is appropriate for children in primary school, and students should learn to develop algebraic thinking from the outset in the school system [4,11]. The integration of algebraic reasoning into the early-year education system will give students the opportunity to develop deeper and more complex cognitive development [8] and help students think algebraically while learning arithmetic [15,16]. Therefore, this study focused on how students' relational thinking while they are learning arithmetic.

As according to [15] relational thinking has role as a bridge between number and number operations and early algebra reasoning. One of the five central concepts supporting this theoretical position is "number sentence structure" [12,14,15]. The meaning of relational thinking as an expression of arithmetic is also the basis for learning algebra, or is known as algebraic reasoning [15].

As far as algebra is concerned, we can characterize relational thinking as how we view equations and expressions holistically, paying attention to the number of relationships within and between these equations and expressions by analyzing the structure of mathematics and key elements of the sentence to produce productive solutions[14,17]. In particular, [14] explained that relational thinking represents an approach to working with numbers that differed from a computational process in a single step-by -step sequence. [18] described algebraic thinking through the 47 + 25 operation. The number operation can be converted to 50 + 22 by adding 3 to 47 and subtracting 3 from 25. This method is called a method of compensation and equalization. They argued that to think this way, students must first be able to identify a significant number relationship by looking at the original number sentence as a whole and at least be implicitly aware of number operation properties.

Carpenter TP, Franke ML. Stephens M refer relational thinking, specifically to the student perspective on equal sign [19–21]. They explained that if students see an equal sign as a symbol of a relationship, it may involve an awareness of the structure of expression and a sensible strategy to solve the numerical sentence according to the operations involved. However, [15] argued that equality in relational thinking must be combined with other concepts of relational thinking, including the structure of numbers. In detail, [14] explained three specific applications of relational thinking: 1) Viewing an equal sign as an indicator of a relationship, 2) Simplifying the calculation by reference to the relationship between numbers, 3) Making general relation explicitly based on the fundamental characteristics of the number operations.

Based on these three specific applications of relational thinking explained by [14], this study focused on identifying the students' relational thinking encouraging through Flow of Lesson in classroom implemented Lesson Study and Open Approach. Flow of Lesson is the idea put forward by [22] as linkages between real-world students and real-world mathematics and a tool for accessing student relational thinking when students are involved in problem-solving processes. As we know students in grade one much more familiar with their own real world and otherwise too hard for them to understand mathematics world. Therefore, by facilitating students to catch up real-world of mathematics using students' real world was wished students' relational thinking can be encouraged and their thinking process also become easier to be accessed by observers.

While, Lesson Study and Open Approach are two innovations in the teaching process proposed by [23] to change the paradigm of teaching practices of teachers and to improve teaching practices on a consecutive basis. Open Approach is a teaching approach that emphasizes individual differences in the thinking of each student. This teaching approach contains four steps: 1) Teacher Open-ended problem to students; 2) students learning by themselves while they are solving problem; 3) Doing whole class discussion; 4) Teacher connect students' mathematics ideas emerged in the classroom. We are expected that by using this teaching approach, the differences the way of students' thinking can be catch up and be identified then become the valuable lesson for mathematics educators and researchers.

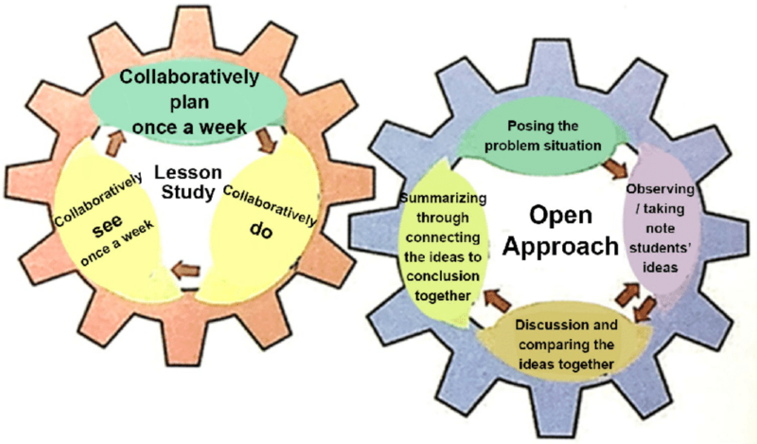

Figure 1 The Weekly Cycle of Lesson Study and Open Approach

Moreover, Lesson Study is teachers' professional development by working collaboratively to improve and develop Open Approach directly [23] having 3 phases: 1) Collaboratively Design Research Lesson (Plan), 2) Collaboratively Observe Research Lesson (Do) and 3) Collaboratively Reflect on Teaching Practice (See). The integration of these teaching innovations is shown in Figure 1. Therefore, this research focused on students' learning tools and teacher development to get a great students' relational thinking.

Research methodology

Qualitative method was used in this research to analyze students' relational thinking deeply. This qualitative research involved participatory ethnography, in which researchers participated as a member of the Lesson Study team. This Lesson Study team collaboratively to collect data, observe classroom activity and interview students directly in classroom activities. Lesson Study team was consisted of: two Grade One in-service teachers, one pre-service teacher, the researcher

While the phases of the Lesson Study to work collaboratively explained as follows:

1. Collaboratively to plan, the lesson plans focus on where the student's relational thinking can be developed by using the lesson flow for three specific applications of relational thinking.

2. Collaboratively to observe, the classroom activity focused on students' awareness of the structure of number sentences involving the relationship between numbers and the fundamental characteristics of the number operations in which one member of the team practiced teaching.

3. Collaboratively to see, the teaching and learning activities focused on how the relationship thinking of students could emerge through the flow of lessons.

Data Collection

Data collection was conducted in the Khu Kham Phithaya San School Grade 1 Mathematics class, Sam Sung District, Khon Kaen Province. This school has used the innovations Open Approach and

Lesson Study. The school is eager to develop and collaborate on the continued implementation of Lesson Study and Open Approach by providing opportunities for government agencies and various organizations to observe and learn about the use of Lesson Study and Open Approach in which teachers and students are familiar with outsiders. The data collection process was carried out during the 2019 academic year in which the researcher worked collaboratively in a weekly cycle with the other Lesson Study members at every step of the Lesson Study, integrated with the Open Approach phases, and used Japanese mathematics textbooks in the Thai version, highlighting the relationship thinking of the students. The research instrument in this study is detailed as follows:

1. Audio and video audio recordings are used to conjunction with teaching and learning processes in the classroom.
2. The written works of the students.
3. Field notes taken by a member of the teaching study team, which focused on students ' awareness of the structure of the number of sentences, including the relationship between numbers and key characteristics of number operations.

Research Results

These research results based on empirical evidence collected in the mathematics classroom using the Lesson Study and Open Approach. Those shown that the relational thinking of students emerged when students were aware of the structure of number sentences involving the relationship between numbers and the fundamental properties of number operations through the flow of lesson in three specific applications of relational thinking. On detail, researchers explained how Flow of Lesson can help students to develop their relational thinking in addition and subtraction operations.

Real-world representation helps students to understand the relationship between numbers and the fundamental properties of number operation

Giving students real world situation make them can imagine the problem then solved it by their experiences. Before the class, members of Lesson Study team discussed to design real world situation. As the principle of Open Approach, Lesson Study teams have to design real world situation which is can stimulate students to solve the problem by themselves and contain more than one solution. Moreover, member of Lesson Study team must predict the students' miss-understanding while they are given the situation, as well as what teacher should do to anticipate those. In this step, the competence of Lesson Study team is very influential. In this section, members of Lesson Study team gave students situation: The teacher went to Nong Khai by taking the train to There were 9 people on the train. At first, there were 5 people walking into the train. Then in the next station 7 people got off the train. Students were asked to count how many people are left on the train. Lesson study team hope through this real-world situation, students can understand the relationship between numbers and can understand the properties of number operation: addition and subtraction.

This conversation was happening in the class. It shows how real-world representation helps students to understand the relationship between numbers and the fundamental properties of number operation. According to item 4, with the help of real-world representation, student 2 can realize that the keyword is "people got off" so the question should be "how many people are left on the train" and the operation were addition and subtraction. This case was an indicator for students' relational thinking of how numbers are interconnected and how numbers can be used in meaningful ways then generally make sense of arithmetic (14).

Table 1 Students' Conversation in Real-World Representation Part

Item: 1	Teacher	Right now, it's 14 people, right? Now the train keeps going. Arrive at the next station, 7 people got off the train
Item: 2	Student 1	Teacher, is it addition or subtraction?
Item: 3	Teacher	What do you think the teacher would ask?
Item: 4	*Student 2*	*How many people are left on the train?*
Item: 5	Teacher	Oh, my friends. How many people are on the train?
Item: 6	Student 3	7 people left
Item: 7	*Student 2*	*Addition then subtraction*
Item: 8	Teacher	Oh, I will repeat the story again so I won't forget what our situation is.
Item: 9	student	9 children are on the train. There are 5 more children on the train and at the next station, 7 are off the train. How many children are on the train now?

Furthermore, according to [24] students need to focus on relations between minuend and subtrahend in order to make them see arithmetic not as a set of rules. Otherwise, it is a necessity for students to see the relations underlying the rules and to develop fundamental arithmetic skills [25]. Unfortunately, at this section the students understanding about the relation between the operations not clear enough and we did not see students' understanding for the relation among numbers and these became the focus of lesson study team for the next lesson.

Semi-concrete aids help students to reason number relation and fundamental properties of number operation

This lesson is included in the application of the student's relational thinking to the use of number relationships to simplify calculations. As the reflection for the previous lesson, students cannot see the relation among numbers, those Lesson Study team give other situation in this section. The situation was designed so that students can reason the relations among numbers clearly. At the first time they learn about number, particularly about number decomposition, students use base-ten block. Their understanding about number decomposition using base-ten block made them fluently in decomposing numbers using diagram while they solve addition and subtraction operations. Therefore, semi-concrete aids used in this section is diagram.

Table 2 Students' Conversation in Semi-Concrete Aids Part

Item: 10	*Student 4*	*9 + 5-7 … 5 Let 9 go 1 to 10 …. 5 left 4 …. 10 combined with 4 is 14 … 14-7 … 10 pulled down 7 … 4 left 4 … 10 left 3 …. 3 and 4 the total together is 7*
Item: 11	Teacher	Okay, give a little applause for my friends. Very good. At first, what do friends do?
Item: 12	student	9 + 5
Item: 13	Teacher	Take 9 + 5 first, right? 9 + 5 What does this friend do?
Item: 14	student	5 give 9 go 1
Item: 15	Teacher	What would it be 10?
Item: 16	student	Get 9 into 10 first
Item: 17	Teacher	Oh, make 9 into 10 before finally getting 14. What to do next?
Item: 18	student	Remove
Item: 19	Teacher	Oh, 14-7 friends. What to do?
Item: 20	student	Take 10, pull down 7

According to table 2, students seemed flexible in decomposing numbers. Their way of decomposing numbers not only decomposes a number into a few smaller numbers, but implicitly what they do makes sense. Starting with the first idea (item 10), he took 1 out of 5 to be added to 9 because he wanted to make 10 (number relation to 10), and he knew that 6 and 4 would build 10 (part-total relationship). Then continue the subtraction operation, $14 - 7$, as the beginning of which decomposed the number 14 into 10 and 4 then took 7 as a subtraction number out of 10 or, mathematically, we can write its calculation process as $9 + 5 - 7 = [9 + (1 + 4)] - 7 = [(9 + 1) + 4] - 7 = (10 + 4) - 7 = 14 - 7 = (10 + 4) - 7 = (10 - 7) + 4 = 3 + 4$. [10] points out two indicators for students' capability in relational thinking: students are able to restructure arithmetic operations to change the given calculation and to transform the number sentences with the use of fundamental arithmetic properties. Implicitly, it appears that while student decomposed numbers. They applied the associative property in addition to the subtraction operation, and used a diagram which helps them to do so more flexibly. Even though some of them haven't decomposed numbers into a few smaller numbers, and this became the point to be considered by member of Lesson study team for the next class.

Mathematics representation helps students to reason the number relationship and the fundamental characteristics of the number operation in the general case

In this section, the number operation created by the students according to the real-world representation given by teacher. As the reflection from the previous class, the members of Lesson Study team gave students chance to explain their idea while they add and subtract numbers by decomposing numbers. This aims to know the way of student reasoning about the relations among numbers and the fundamental characteristics of number operation in the general case. Another goal is to repair students' miss-understanding particularly about the case in previous lesson.

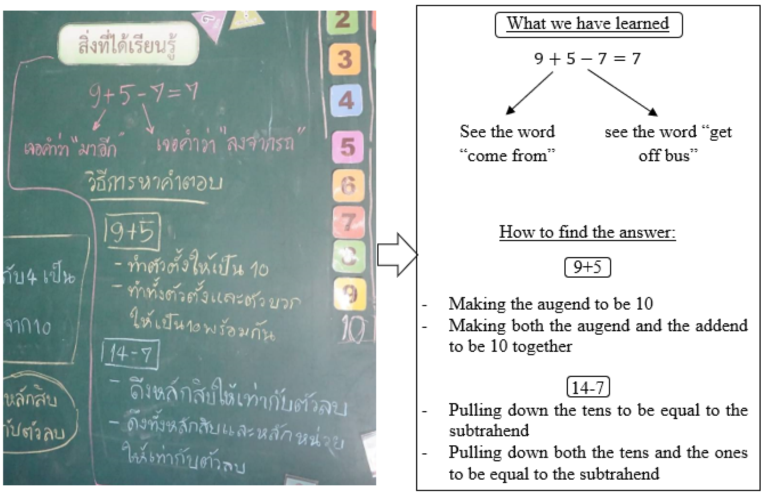

Figure 2 First Students' Idea by Using Semi-Concrete Aid in The Sixth Lesson In Thai (Left) And Was Translated To English (Right)

According to the picture data and video recording Lesson study team analyze how class was going on and how students though along the class activity. To create number operation, students changed the real-world representation to a representation of the mathematical world, helped by semi-concrete aid. Then according to mathematics representation, students used it to explain both the number relationship and the fundamental properties of the number operation in the general case. Figure 2 shows learning conclusion arranged by students and helped by teacher. In the fourth section of Open Approach, students are gotten used to recall what they have learned at the day. According to figure 2, students have reason where "+" and "–" come from. In other word they know when they have to add or subtract. This indicates students' ability in reasoning the relationship between numbers and fundamental mathematics operation [26].

Moreover, to add 9 to 5, they making the augend to be 10 first or making both the augend and the addend to be 10 together. Likewise, in subtraction, beside of pulling down the tens to be equal to the subtrahend, they also can pull down both the tens and the ones to be equal to the subtrahend.

This tells us that because of their good ability in relational thinking, they used it to decompose numbers flexibly to get more than one way in solving a calculation.

Table 3 Students' Conversation in Mathematics Representation Part

Item: 21	Teacher	Take 9 + 5 first, right? 9 + 5 What methods can be done?
Item: 22	*student*	*Set the number to 10 first.*
Item: 23	Teacher	What about the methods of the next group?
Item: 24	student	Set your top and bottom numbers to be 10 at the same time.
Item: 25	Teacher	Next, what do we do next? 9 + 5, what are we doing next?
Item: 26	student	Delete
Item: 27	Teacher	What, delete anything
Item: 28	student	14-7
Item: 29	Teacher	What methods can you do?
Item: 30	student	Can be pulled from both directions
Item: 31	Teacher	How is the group 2 method?
Item: 32	*student*	*Pull the tens place exactly equal to 7*
Item: 33	Teacher	Did he called pulling from the tens place which is equal to which one?
Item: 34	student	Eraser

Those evidence become stronger while the members of Lesson Study team analyze students recording conversation. According to item 22, for additional operation in general, students said that we had to think in relation to constructing number 10 by any means in decomposing numbers, then subtraction (item 32), they focused on how to think in relation to constructing subtrahend number by any process for decomposing numbers, without mentioning role 0 neither in addition nor subtraction. Even though they applied it in simple terms.

Equal sign

This lesson was a student's examination of the concept that they learned about addition and subtraction. According to the previous classes, the students' relational thinking is stronger day by day. Then in this lesson, Lesson Study team design the quick examination to see their thinking deeply. Again, in this step, observer accuracy is very needed. Observer were needed to catch up students' idea while they speak together and choose some students who have good idea to be presented. Because, at this lesson students were given some number operations then counted the result together. Then the teacher asked the students what could be seen from the number of sentences. The student went directly to the board to arrange the numbers, such as in Figure 3, and the teacher asked the other students again what could be seen from the number of sentences arranged by their friend.

Figure 3 Student Arranging Number Sentence Using Relational Thinking

At the end of the class Lesson study team consider the students’ conversation through audio-video recording, a great-evidence for the students' rationale for the number relationship has appeared in this lesson. According to students’ conversation in table 4, students not only concern on finding the result of operations, but they also focus on examine the relations between the given quantities. This thinking belonging to the properties of relational thinking (24). Although the problem was directly addressed to the students as a mathematical representation, their understanding that was built through the Flow of Lesson applied in every previous lesson is very great. Considering the study by (27), they found that the student can see the relationship between numbers after the number sequence was arranged by the teacher, but in this class, they arrange the number sentences on their own and then explain the relationship between them on their own. In addition, this number relationship has also been explicitly generalized. Of course, this is the ability of students, which is only view students in Grade One elementary school have this ability. According to this lesson, we found that the students’ habit of identifying the number relationship through Flow of Lesson made it easier to reason and generalize the number relationship, although it was presented directly in mathematical representation, which was helped by the arrow as their semi-concrete aids.

Table 4 Students' Conversation in equal sign part

Item: 35	Teacher	Has anyone else noticed anything? No more, right? Give a round of applause Now let's observe the subtraction card See what
Item: 36	student	My teacher, number bellows, saw him falling.
Item: 37	Teacher	Oh, my friend said that the number has decreased.
Item: 38	Teacher	So, what is the negative number?
Item: 39	student	The subtracted number is similar.
Item: 40	Teacher	How is the answer?
Item: 41	student	Answer increased
Item: 42	Teacher	Which, see the answer. 15-7 How much?
Item: 43	student	8
Item: 44	Teacher	14-7 How much is the answer?
Item: 45	student	7
Item: 46	Teacher	Then look at the answer
Item: 47	student	Answer in order
Item: 48	Teacher	How to arrange
Item: 49	student	It decreases
Item: 50	Teacher	Friends said the answer also decreased.
Item: 51	student	If the number decreases, the answer decreases.
Item: 52	Teacher	When the negative numbers remain the same?
Item: 53	student	Same
Item: 54	Teacher	If the minus is the same, it's OK.

Conclusion and Discussion

Results reflecting evidence in mathematics classrooms using the Lesson Study and Open Approach have shown that the flow of lessons can be provided as assistance to teachers who work collaboratively 1) plan 6 lessons that work together to develop open-ended problems, design materials and equipment, anticipate students ' mathematical ideas as well as respond to open-ended learning. In their collaborative work, they have created a dynamic process of intellectual development for teaching and learning. They not only follow key elements of collaborative learning, but also focus on the use and sharing of sample teaching lessons or specific classroom contextual issues as opportunities for reflection-in-action to foster collaborative discussions among group members and to further engage in reflection-on-action.

In detail, we have found that real-world representation can help students to understate the relationship between numbers by focusing on the specific keyword used in an open-ended problem posed by a teacher in a problem situation. This is in line with studies by [28,29] which found that in mathematics Word Problem, for any verbal formulation (modeling) that could be used, students must underlie the number relationships that can be established by focusing on the specific keyword used in the problem. Then, aided by the diagram as a semi-concrete aid, students became easier to explain the structure of the number sentences: reasoning the relationship between numbers and reasoning at the same time the fundamental properties of number operations, making them easier and more flexible to decompose numbers. This is similar to what is explained by [14] students who think relationally can explain which transformations make sense in a particular problem they need to understand, at least implicitly, the relationship between numbers and the fundamental properties that can be used to justify transformations of expressions. The last mathematical representation, based on real-world representation, helped by semi-concrete aids, made it easier for students to reason the structure of number sense in the general case. So, we have shown that the flow of lessons [22] in the classroom, which implement Lesson Study and Open Approach, can develop the relational thinking of students, helping them to learn algebra in the upper class.(30)

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References

1. NCTM. Principles and Standards for School Mathematics. Reston, VA: National Council of Teachers of Mathematics; 2000.
2. Carpenter TP, Levi L. DEVELOPING CONCEPTIONS OF ALGEBRAIC REASONING IN THE PRIMARY GRADES. 2000; (002).
3. Kaput JJ. Transforming Algebra from an Engine of Inequity to an Engine of Mathematical Power by “Algebrafying” the K-12 Curriculum. In: National Council of Teachers of Mathematics, editor. The Nature and Role of Algebra in the K-14 Curriculum: Proceedings of a National Symposium. Washington, DC: National Academy Press; 1998. p. 25–6.
4. Stacey KC, Chick H. Solving the Problem with Algebra. In: Stacey KC, editor. The Future of the Teaching and Learning of Algebra: The 12th ICMI Study. Dordrechtt, The Netherland: Kluwer Academic Publishers; 2004. p. 1–20.
5. Usiskin Z. Conceptions of School Algebra and Uses of Variables. 1999; 7–13.
6. Chimoni M, Pitta-pantazi D, Chimoni M, Connections DP, Chimoni M, Pitta-pantazi D. Connections between algebraic thinking and reasoning processes To cite this version : HAL Id : hal-01286897 Connections between algebraic thinking and reasoning processes. 2016.

7. Stephens AC, Fonger N, Strachota S, Isler I, Blanton M. A Learning Progression for Elementary Students' Functional Thinking. Math Think Learn [Internet]. 2017; 19(3): 143–66. Available from: https://doi.org/10.1080/10986065.2017.1328636
8. Blanton ML, Kaput JJ. Characterizing a Classroom Practice That Promotes Algebraic Reasoning. J Res Math Educ. 2005; 36(5): 412–46.
9. Carraher DW, Schliemann AD, Brizuela BM, Earnest D. Can Young Students Operate on Unknowns? J Res Math Educ. 37: 87–115.
10. Kieran C. Algebraic thinking in the early grades: What is it. Math Educ. 2004; 8(1): 139–51.
11. Lins R, Kaput JJ. The Early Development of Algebraic Reasoning: The Current State of the field. In: K. C. Stacey, editor. The Future of the Teaching and Learning of Algebra: The 12th ICMI Study. Dordrecht, The Netherland: Kluwer Academic Publishers; 2004. p. 47–70.
12. Cai J, Lew HC, Morries A, Moyer JC, Ng SF, Schmittau J. The Development of Students' Algebraic Thinking in Earlier Grades: A Cross-Cultural Comparative Perspective. ZDM - Int J Math Educ. 2005; 37(1): 5–14.
13. Carraher DW, Schliemann AD, Brizuela BM, Earnest D. Arithmetic and algebra in early mathematics education. J Res Math Educ. 2006; 37(2): 87–115.
14. Jacobs VR, Franke ML, Carpenter TP, Levi L, Battey D. Professional development focused on children's algebraic reasoning in elementary school. J Res Math Educ. 2007; 38(3): 258–88.
15. Stephens M, Ribeiro A. Working Towards Algebra: The Importance of Relational Thinking. 2012; 15(3): 373–402. Available from: www.redalyc.org/articulo.oa?id=33524579006
16. Napaphun V. Relational Thinking : Learning Arithmetic in order to Promote Algebraic Thinking. J Sci Math. 2012; 35(2): 84–101.
17. Molina M, Castro E, Ambrose R. Enriching Arithmetic Learning by Promoting Relational Thinking. Int J Learn Annu Rev. 2006; 12(5): 265–70.
18. Stephens M, Wang X. Investigating some junctures in relational thinking: A study of year 6 and year 7 students from australia and china. J Math Educ. 2008; 1(1): 28–39.
19. Carpenter TP, Franke ML. Developing algebraic reasoning in elementary school: Generalization and proof. In: K. S. H. Chick, editor. Proceedings of the 12th ICMI Study Conference. Melbourne: University of Melbourne; 2001. p. 155–62.
20. Stephens M. Students' emerging algebraic thinking in the middle school years. In: Watson J, Beswick K, editors. Mathematics: Essential Research, Essential Practice, (Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia. Adelaide: Merga; 2007. p. 678–87.
21. Stephen M. Some key junctures in relational thinking. In: Goos M, Brown R, Makar K, editors. Navigating currents and charting directions Proceedings of the 31st annual conference of the Mathematics Education Group of Australasia. Brisbane: Merga; 2008. p. 491–8.

22. Inprasitha M. Open Approach Lesson Study as an Innovation for Teaching Mathematics. In: APEC-Tsukuba and UNESCO(MGIEP) International Conference XII: Innovation of Mathematics Education through Lesson Study Textbook Development for SDGs, STEM, and Energy by Cross-border Education. Tokyo: Criced.tsukuba; 2018.
23. Inprasitha M, Isoda M. Mathematics for Elementary School: Grade 1. Khon Kaen: Klangnana Vitaya; 2011.
24. Kızıltoprak A, Köse NY. Relational thinking: The bridge between arithmetic and algebra. Int Electron J Elem Educ. 2017; 10(1): 131–45.
25. Knuth E, Stephens AC, Blanton ML, Gardiner AM. Build an Early Foundation for Algebra Success. Phi Delta Kappan. 2016; 97(6): 65–8.
26. Carpenter TP, Levi L, Franke ML, Zeringue JK. Algebra in Elementary School: Developing Relational Thinking. ZDM - Int J Math Educ. 2005; 37(1): 53–9.
27. Stephens M, Armanto D. How to Build Powerful Learning Trajectories for Relational Thinking in the Primary School Years. Shap Futur Math Educ Proc 33rd Annu Confereance Math Eduction Res Gr Australas. 2009; 523–30.
28. Paige JM, Simon HA. Cognitive Processes in Solving Algebra Word Problems. In: Kleinmuntz B, editor. Problem Solving. John Wiley & Sons; 1966. p. 51–5.
29. Nesher P, Teubal E. Verbal cues as an interfering factor in verbal problem solving. Educ Stud Math. 1975; 6(1): 41–51.
30. Stephens AC. Equivalence and relational thinking: Preservice elementary teachers' awareness of opportunities and misconceptions. J Math Teach Educ. 2006; 9(3): 249–78.