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Stock Price Forecasting: Geometric Brownian Motion and Monte Carlo Simulation Techniques

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ABSTRACT

This research paper aims to explore, compare and evaluate the predictive power of the Geometric Brownian Motion (GBM) and the Monte Carlo Simulation technique in forecasting the randomly selected 10 listed stocks in the SET50 of the Stock Exchange of Thailand (SET). The results shows that for the highest precision +/-0.5% of predicted 45 days return, the percentage of accuracy is at the highest of around 5% (or 500 times in 10,000 trials) for both GBM and Monte Carlo Simulation. It can be concluded that model accuracy in predicting end period returns is limited. Especially, predictive power of the models are declining towards the longer the evaluated timeframe. Comparing GBM and Monte Carlo Simulation in term of percentage of accuracy in predicting the end period returns, the two techniques are indifferent. For the predictive power of movements in prices, the GBM is a preferred technique. Besides, Monte Carlo Simulations yields a better accuracy especially in a longer period of evaluated timeframe. In conclusion, both techniques can predict stock prices within a highly accurate range. Thus, the techniques can be applied for stock price forecasting with limits mentioned.

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Keywords: Stock Price, Forecasting, Geometric Brownian Motion, Monte Carlo Simulation

Introduction

Stock price forecasting is widely accepted as a supporting tool for investment decision and investment risk management, later briefly be discussed in the selected list of literatures. There are extensive uses of statistical modeling in stock price forecasting both in academia and business world. General aims of models are to forecast the movement of price, return, volatility, changing in trend, or exploring relationships with other financial assets, business or economic data. Reviews on this matter, selected by number of literatures, can be seen far back since Patell (1976), Bollerslev (1987), Amihud (2002), Ser-Huang and Granger (2003), Pai and Lin (2005), and Chang and Liu (2008). As seen in the selected works mentioned, there are many sophisticated techniques employed in fitting the actual and forecasted data. This paper explores one of the techniques namely Geometric Brownian Motion (GBM) in predicting stock prices. Also, the predictive power of the model will also be evaluated against the criteria, Mean Square Error (MSE), Mean Absolute Percentage Error (MAPE), and selected alternative method i.e. a simple Monte Carlo Simulation. Data used in this research are drawn from 10 randomly selected SET50 stock prices in a range of one trading years (245 days).

Research Objectives

1. Evaluative the forecasting accuracy of using GBM in forecasting selected Thai stock prices
2. Compare the predictive power of that of GBM with the alternative method, simple Monte Carlo Simulation

Expected Benefits

This research work tries to answer whether the results from the GBM prediction may help investor predicting prices, hence reducing investment risk in a specific period of time or not. This may help investors as one of the decision tools in modern day investment with big data and growing computing capacity.

Literature Reviews

In forecasting movement of stock prices, Geometric Brownian Motion (GBM) is a mathematical technique exhibiting the fact of stochastic movement of stock prices. The technique was popularized being employed in option pricing in Black–Scholes model. A detailed work explaining the formulization of Brownian Motion and the Black-Scholes model can be seen in Prabakaran (2014). GBM is also being widely used modeling of stock price. For simplicity, it is assumed that there are two main components of the price movement, i.e. (average daily) price drift and (average daily) volatility (Reddy and Clinton (2016)). The average price drift for each stock (μ) is the mean of continuous compound percentage change in closed price during the time interval can be calculated as followed:

$$\mu = \frac{\sum_{i=1}^T \ln\left(\frac{s_t}{s_{t-1}}\right)_i}{T} \quad (1)$$

where: μ is the average price drift, s_t and s_{t-1} are closed stock price at time t and $t-1$ (or the prior period) respectively, and T is the time interval in days.

Meanwhile, the average daily volatility is calculated from standard deviation (σ) of the continuous compound percentage change in daily closed price ($\ln\left(\frac{s_t}{s_{t-1}}\right)$).

For the definition of returns of the stock price from GBM, the returns are assumed to be log-normally distributed at mean of $(\mu - \frac{\sigma^2}{2})T$ and standard deviation of $\sigma\sqrt{T}$ or:

$\ln\left(\frac{s_t}{s_{t-1}}\right) \sim \phi\left[\left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right]$. Hence, the GBM simulated stock price can be calculated

from the returns above with two main components i.e. the deterministic component or mean of the daily drift scaled by T (or $(\mu - \frac{\sigma^2}{2})T$) and the stochastic component of the standard deviation $\sigma\sqrt{T}$ assuming the random volatility of $\varepsilon \sim N(0,1)$.

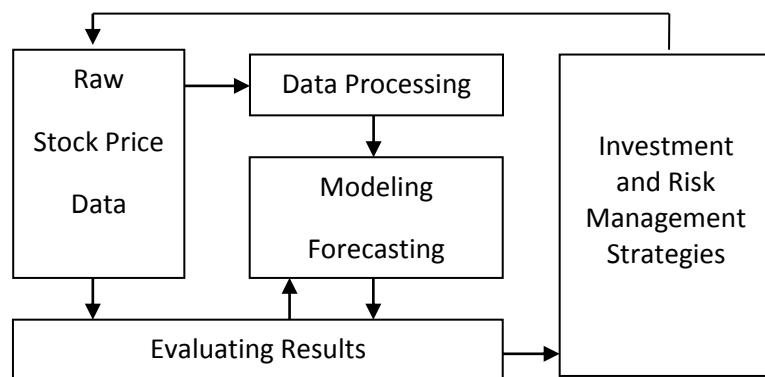
Abidin and Jaffar (2012) applied the GBM to forecast future stock prices for the short-term investment in case of 24 Malaysian stocks. It was found that the results of the model was suitable for the short-term investment (a maximum of two week). The advantage of GBM was a comparatively simple calculation considering other forecasting techniques. Recently, Reddy and Clinton (2016) employed GBM in simulating 50 Australian stock prices. Using Capital

Asset Pricing Model (CAPM) to estimate expected annual return, and standard deviation of the daily return of stock price for the empiric volatility in the simulation, it was found that over all time horizons the chances of a stock price simulated using GBM moving in the same direction as real stock prices (little greater than 50 percent).

Nevertheless, for stock price forecasting Thailand case has also been explored employing other techniques. For example, Kawinpas et al (2015) utilized Bayesian Network and Exponential Smoothing Holt method in predicting Thailand property and construction stocks' price index (monthly data, during the period of January 2009 to December 2013). Using lowest mean square error criteria, the results showed that the model best predicting period was around 1 to 3 months. Recently, Dhakonlayodhin and Areepong (2018) employed Moving Average (ARIMA) and Autoregressive Integrated Moving Average with Exogenous Variable (ARIMAX) model in forecasting individual monthly stock price, namely Bangkok Bank Plc. during the period of January 2012 to December 2016. Findings suggested that, in predicting 12-month-prices of the stock, the ARIMAX model was the most efficient one by providing the minimum Mean Absolute Percentage Error (MAPE) and Root Mean Squared Error (RMSE).

Research Framework

From the review, aforementioned, this research framework can be shown as a figure below:



From the figure above, the main research question is that whether or not the Geometric Brownian Motion (GBM) technique can properly predict stock price comparing with the Monte Carlo Simulation technique. Raw stock price data are gathered and processed align

with the techniques employed. Then the selected model can be tested against the criteria and/or other choice(s) of model. The results are then can be used in making investment or risk management decisions. Hence, the activities goes on as a loop in improving the data processing and stock price forecasting techniques.

Research Methodology

The simulation techniques use in this paper are the Geometric Brownian Motion (GBM) and Monte Carlo Simulation technique as details mentioned in the review part. The ten sampling stock prices are randomly picked from stocks listed in the SET50 in the Stock Exchange of Thailand. The table below are the stock initial details.

Table 1: 10 Randomly Selected SET50 Companies

Company	Symbol	Sector
Advanced Info Service	ADVANC	Information & Communication Technology
Airports Of Thailand	AOT	Transportation & Logistics
Bangkok Bank	BBL	Banking
Berli Jucker	BJC	Commerce
Charoen Pokphand Foods	CPF	Food & Beverage
Global Power Synergy	GPSC	Energy & Utilities
Home Product Center	HMPRO	Commerce
Intouch Holdings	INTUCH	Information & Communication Technology
Kiatnakin Bank	KKP	Banking
Thai Union Group	TU	Food & Beverage

Note: SET50 List as of 1st January to 30th June 2019

From the randomly selected stocks above, the adjusted price data of the sampled stocks are retrieved from Yahoo Finance during the period of March 2018 to March 2019, a total of 245 days. Then, the average price drift for each stock (μ) is the mean of continuous compound percentage change in closed price during the time interval can be calculated as mentioned earlier. In this case T or the time interval in days is one trading year (245 days). The period is divided in two parts: information from the first part of the year (200 days = T) is for calculating all needed parameters for the simulations, and the second part (45 days) is for evaluating simulation results.

For the empiric average daily volatility, the data is calculated from standard deviation (σ) of the continuous compound percentage change in daily closed price. Therefore, the

GBM simulated stock price can be calculated from the two main components i.e. the deterministic component or mean of the daily drift scaled by T (or $(\mu - \frac{\sigma^2}{2})T$) and the stochastic component of the standard deviation $\sigma\sqrt{T}$ assuming the random volatility of $\varepsilon \sim N(0,1)$.

For the simple Monte Carlo Simulation, the continuous compound percentage changes in closed prices in daily basis are calculated and indexed. Then the indexed percentage changes in daily stock price will be randomly selected in calculating the simulated prices. Therefore, $s_t = s_{t-1}e^\gamma$ assuming γ is a randomly selected return from the set of existing percentage changes in closed prices from the prior period (in this case the returns are the prior 200 daily returns).

To evaluate the models forecasting results, for each stock, 10,000 cases of simulation for both models are made. Then the average forecasted price for each stock will be evaluate against the real prices. Mean Square Error (MSE) is used as a evaluation criteria as stated below:

$$MSE = \frac{\left[\sum_{t=1}^T (s_t - \hat{s}_t)^2 \right]}{T} \quad (2)$$

where \hat{s}_t is the forecasted average stock price and T is the time interval for evaluation (45 days in this case).

Additionally, for evaluating the model forecasting accuracy in a given period of time, in this case 7 and 14 days, Mean Absolute Percentage Error-MAPE for each stock is employed. The formula and criteria can be summarized below.

$$MAPE = \frac{\sum_t^T \left| \frac{s_t - \hat{s}_t}{s_t} \right|}{T} \quad (3)$$

where \hat{s}_t is the forecasted stock price at time t and T is the time interval for evaluation (7 and 14 days in this case). The accuracy criteria cab be seen below.

Table 2: Accuracy Criteria

MAPE	Judgment of Forecast Accuracy
<10%	Highly Accurate
11% to 20%	Good Accurate
21% to 50%	Reasonable Forecast
> 51%	Inaccurate Forecast

Source: Abidin and Jaffar (2012)

Research Results

From the methodology stated above, 10,000 simulations of each 10 randomly selected SET50 stock prices are performed for a period of 45 days. For illustration purpose, 100 scenario for ADVANC from GBM and Monte Carlo Simulations are shown below:

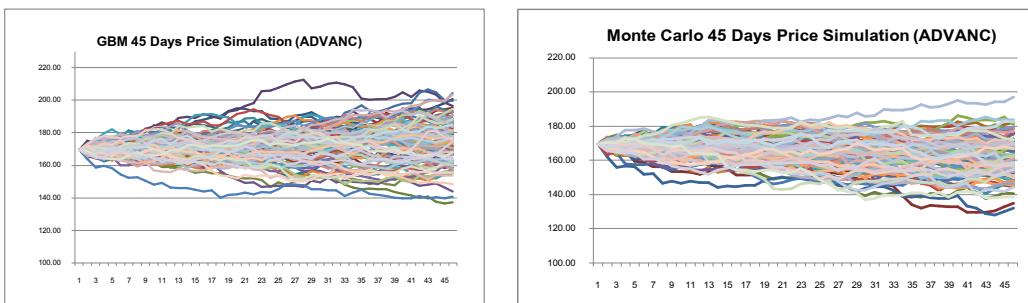


Figure 1: Price Simulation Results (ADVANC)

Source: Authors Calculation

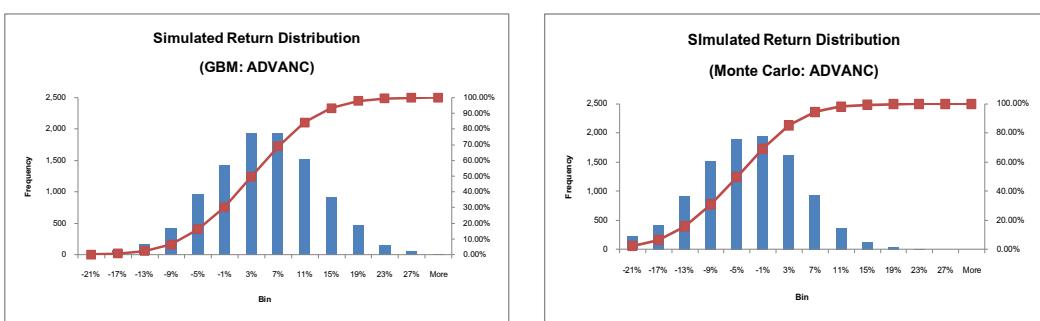


Figure 2: Simulated Return Distribution (ADVANC)

Source: Authors Calculation

Figure 2 exhibits simulated return distribution from 10,000 simulated cases both for the case of GBM and Monte Carlo Simulation. It can be seen that results from GBM show more optimistic returns. Such simulated return distribution above can be used as a tool in managing investment risk especially Value at Risk (VaR), allowing investor to prepare for probability of losses. Hence, the predictability power of the models in term of accuracy in forecasting ending period return (in this case 45 days) is essential.

Table 2 shows the percentage of accuracy (out of 10,000 trials) in forecasting real rate of return for each stock in various ranges i.e. +/-0.5%, +/-1.5%, and +/-2.5%. It can be seen that, for the highest precision +/-0.5% of predicted 45 days return, the percentage of accuracy is at the highest of around 5% (or 500 times in 10,000 trials) for both GBM and Monte Carlo Simulation in case of BBL. Allowing 5% or +/-2.5% error range of the predicted 45 days return, the percentage of accuracy is then being increased to the highest of around 26% (or around 2,600 times in 10,000 trials) for the case of BBL as well. From the 10 sampled stocks, it can be concluded, in this part, that in term of percentage of accuracy in predicting the end period returns, results from the GBM and Monte Carlo simulation are generally tie.

However, turning to predictive power of movements in prices, considering MSE from the average price of 10, 100, 1,000, and 10,000 simulations, detailed in Table 2, the GBM shows clear superior results.

Table 2: Detailed Results

Simulation Details	ADVANC		AOT		BBL	
	GBM	Monte Carlo	GBM	Monte Carlo	GBM	Monte Carlo
Drift (200 Days)	15.023%	NA	-5.938%	NA	0.352%	NA
Volatility (199 Days)	17.028%	NA	17.454%	NA	15.639%	NA
Drift (Daily)	0.075%	NA	-0.030%	NA	0.002%	NA
Volatility (Daily)	1.21%	NA	1.24%	NA	1.11%	NA
Drift (Mean)	0.07%	NA	-0.04%	NA	0.00%	NA
Real Initial Price (Day 200)	169.40		63.50		205.00	
Real End Period Price (Day 245)	179.50		67.75		206.00	
Average End Period Simulated Price (Day 245)	175.39	161.90	62.64	61.80	205.14	202.52
Real Return (45 Days)	5.793%		6.478%		0.487%	
Average Simulated Return (45 Days)	3.153%	-4.833%	-1.701%	-3.033%	-0.210%	-1.461%
MSE_10	10.23	157.57	22.15	22.25	12.83	121.07
MSE_100	34.12	164.47	20.30	30.02	30.63	47.13
MSE_1000	30.32	162.84	25.08	31.49	21.53	38.62
MSE_10000	26.56	155.97	26.40	31.37	23.78	37.27
+/- 0.5 Percent Accuracy	4.86%	1.97%	2.80%	2.62%	5.02%	5.45%
+/- 1.5 Percent Accuracy	14.20%	6.14%	8.99%	7.53%	15.88%	15.63%
+/- 2.5 Percent Accuracy	23.32%	10.24%	15.17%	12.93%	26.28%	26.26%
MAPE (7 Days) (< 10% = Highly Accurate)	2.20%	6.95%	5.83%	4.99%	1.96%	2.27%
MAPE (14 Days) (< 10% = Highly Accurate)	3.22%	6.36%	7.49%	7.20%	1.91%	4.18%
MAPE (28 Days) (< 10% = Highly Accurate)	2.76%	7.92%	10.94%	7.66%	5.20%	3.27%
MAPE (45 Days) (< 10% = Highly Accurate)	4.61%	9.19%	10.83%	8.89%	5.12%	3.91%

Source: Authors Calculation

Table 2: Detailed Results (Continue)

Simulation Details	BJC		CPF		GPSC	
	GBM	Monte Carlo	GBM	Monte Carlo	GBM	Monte Carlo
Drift (200 Days)	-25.243%	NA	5.603%	NA	-25.616%	NA
Volatility (199 Days)	24.638%	NA	21.870%	NA	26.569%	NA
Drift (Daily)	-0.126%	NA	0.028%	NA	-0.128%	NA
Volatility (Daily)	1.75%	NA	1.55%	NA	1.88%	NA
Drift (Mean)	-0.14%	NA	0.02%	NA	-0.15%	NA
Real Initial Price (Day 200)	47.50		25.00		56.78	
Real End Period Price (Day 245)	50.75		25.75		61.00	
Average End Period Simulated Price (Day 245)	44.94	44.21	25.31	24.80	53.62	52.70
Real Return (45 Days)	6.618%		2.956%		7.177%	
Average Simulated Return (45 Days)	-6.231%	-7.825%	0.682%	-1.276%	-6.518%	-8.196%
MSE_10	14.55	19.50	2.99	3.18	30.63	39.21
MSE_100	9.18	13.94	2.06	3.59	32.06	47.32
MSE_1000	8.21	9.35	2.16	3.04	40.21	49.06
MSE_10000	8.50	11.11	2.31	3.12	40.52	47.88
+/- 0.5 Percent Accuracy	1.64%	1.37%	3.62%	3.37%	1.77%	1.41%
+/- 1.5 Percent Accuracy	5.74%	4.40%	10.96%	11.22%	5.34%	4.63%
+/- 2.5 Percent Accuracy	9.08%	7.95%	18.42%	18.76%	8.90%	8.12%
MAPE (7 Days) (< 10% = Highly Accurate)	4.54%	1.49%	1.05%	1.71%	3.69%	1.16%
MAPE (14 Days) (< 10% = Highly Accurate)	6.09%	2.96%	2.65%	3.73%	5.00%	2.61%
MAPE (28 Days) (< 10% = Highly Accurate)	10.12%	3.31%	4.73%	3.98%	8.86%	8.09%
MAPE (45 Days) (< 10% = Highly Accurate)	11.76%	4.27%	5.54%	3.83%	11.81%	11.01%

Source: Authors Calculation

Table 2: Detailed Results (Continue)

Simulation Details	HMPRO		INTUCH		KKP	
	GBM	Monte Carlo	GBM	Monte Carlo	GBM	Monte Carlo
Drift (200 Days)	5.023%	NA	-15.697%	NA	-6.928%	NA
Volatility (199 Days)	21.873%	NA	13.388%	NA	15.402%	NA
Drift (Daily)	0.025%	NA	-0.078%	NA	-0.035%	NA
Volatility (Daily)	1.55%	NA	0.95%	NA	1.09%	NA
Drift (Mean)	0.01%	NA	-0.08%	NA	-0.04%	NA
Real Initial Price (Day 200)	14.90		46.72		66.25	
Real End Period Price (Day 245)	15.10		54.25		69.75	
Average End Period Simulated Price (Day 245)	15.05	14.84	45.06	44.69	65.16	64.39
Real Return (45 Days)	1.333%		14.943%		5.148%	
Average Simulated Return (45 Days)	0.465%	-0.903%	-3.823%	-4.642%	-1.939%	-3.082%
MSE_10	0.06	0.16	43.67	48.40	14.07	37.87
MSE_100	0.03	0.11	52.04	51.65	9.93	16.84
MSE_1000	0.06	0.11	50.80	54.01	13.20	15.46
MSE_10000	0.06	0.11	50.38	53.79	12.62	15.84
+/- 0.5 Percent Accuracy	3.67%	3.72%	0.07%	0.01%	3.47%	2.66%
+/- 1.5 Percent Accuracy	11.81%	11.62%	0.27%	0.08%	9.96%	8.40%
+/- 2.5 Percent Accuracy	19.41%	19.46%	0.48%	0.12%	17.12%	14.40%
MAPE (7 Days) (< 10% = Highly Accurate)	1.29%	5.05%	1.32%	3.86%	3.60%	2.11%
MAPE (14 Days) (< 10% = Highly Accurate)	1.86%	6.28%	1.93%	3.44%	5.47%	1.93%
MAPE (28 Days) (< 10% = Highly Accurate)	2.87%	6.59%	8.72%	9.47%	6.45%	3.82%
MAPE (45 Days) (< 10% = Highly Accurate)	4.02%	4.57%	13.51%	13.50%	6.06%	7.38%

Source: Authors Calculation

Table 2: Detailed Results (Continue)

Simulation Details	TU	
	GBM	Monte Carlo
Drift (200 Days)	-21.926%	NA
Volatility (199 Days)	22.219%	NA
Drift (Daily)	-0.110%	NA
Volatility (Daily)	1.58%	NA
Drift (Mean)	-0.12%	NA
Real Initial Price (Day 200)	15.75	
Real End Period Price (Day 245)	18.40	
Average End Period Simulated Price (Day 245)	14.96	14.73
Real Return (45 Days)	15.574%	
Average Simulated Return (45 Days)	-5.654%	-7.187%
MSE_10	8.47	8.33
MSE_100	6.67	8.60
MSE_1000	7.21	8.03
MSE_10000	7.27	8.01
+/- 0.5 Percent Accuracy	0.50%	0.41%
+/- 1.5 Percent Accuracy	1.52%	1.11%
+/- 2.5 Percent Accuracy	2.52%	1.86%
MAPE (7 Days) (< 10% = Highly Accurate)	8.34%	4.03%
MAPE (14 Days) (< 10% = Highly Accurate)	13.75%	6.29%
MAPE (28 Days) (< 10% = Highly Accurate)	16.03%	6.51%
MAPE (45 Days) (< 10% = Highly Accurate)	16.15%	7.53%

Source: Authors Calculation

Note: Detailed results are available on request

On the contrary, considering only one trial (the first series of simulated price for each stock), the average MAPEs from the Monte Carlo Simulations show a slightly lower percentage (therefore, better accuracy) especially in a longer period of evaluated timeframe, in this case 7, 14, 28, and 45 days. Results show in Table 3 below. It should be noted that the longer the evaluating period, the higher the MAPE (the lower accuracy). Nevertheless, according to the set criteria, both techniques yield the average MAPE within a highly accurate range of not greater than 10%.

Table 3: Average MAPE for 10 Stocks

Average (10 Stocks)	GBM	Monte Carlo
MAPE (7 Days) (< 10% = Highly Accurate)	3.38%	3.36%
MAPE (14 Days) (< 10% = Highly Accurate)	4.94%	4.50%
MAPE (28 Days) (< 10% = Highly Accurate)	7.67%	6.06%
MAPE (45 Days) (< 10% = Highly Accurate)	8.94%	7.41%

Source: Authors Calculation

Discussion

The advantage of stochastic model of stock price forecast such as GBM and Monte Carlo Simulation is that the simulated return distribution above can be applied not only as an investment decision tool but also a tool in managing investment risk. The techniques allow investor to be able to predict probability or size of losses which may occur. Consequently, the predictive power of the model is a vital. It is shown here, with our sampled stocks, model accuracy in predicting end period returns is limited. Especially, predictive power of the models are declining towards the longer the evaluated timeframe. To compare with the previous works, the results confirm the work of Abidin and Jaffar (2012) which concluded that the GBM suits for short-term investment. For Thailand case, this also echoes the work of Kawinpas et al (2015) concluding that the shorter the forecasting period the better the results. Comparing GBM and Monte Carlo Simulation in term of percentage of accuracy in predicting the end period returns, the two techniques are indifferent. For the predictive power of movements in prices, the GBM is a preferred technique. Besides, Monte Carlo Simulations yields a better accuracy especially in a longer period of evaluated timeframe. In conclusion, both techniques can predict stock prices within a highly accurate range. Thus, the techniques can be applied for stock price forecasting with limits mentioned.

Recommendations

From the results and discussion above, recommendation for investor are both techniques can be applied as investment decision tools knowing that there are limit of the predictive power of the tools. Therefore, more indicators may be utilized in helping the decision making. Additionally, for further research works, alternative models requiring more data and higher computational power exploiting the growing of big data and availability of high computing power both hardware and software should be considered.

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