Kasetsart Journal of Social Sciences 41 (2020) 75–82

Kasetsart Journal of Social Sciences

journal homepage: http://kjss.kasetsart.org

Determination of order pattern frequency analysis using the copula method

Somkid Naprom [a], Kullapapruk Piewthongngam [b,*], Thanaporn Athipanyakul [c,1]

[a] Doctoral program in System Approaches in Agriculture, Faculty of Agriculture, Khon Kaen University, Khon Kaen 40002, Thailand
[b] Faculty of Economics, Khon Kaen University, Khon Kaen 40002, Thailand
[c] Department of Agricultural and Resource Economics, Faculty of Economics, Kasetsart University, Bangkok 10900, Thailand

Article Info

Article history:
Received 11 September 2017
Revised 24 January 2018
Accepted 9 May 2018
Available online 26 May 2018

Keywords:
chicken processing,
chicken size,
copula,
food supply chain,
order frequency and voluminosity

Abstract

The frequency and voluminosity of orders play a crucial role in determining the operation and scheduling, as well as the bird weight to be supplied to chicken processors. Due to the variability in frequency and voluminosity of each product, part, and size, orders have stochastic characteristics in nature; hence, the proper means to investigate orders is using probability theory and the stochastic process method. In this study, copula distributions were exploited to incorporate chicken order features into frequency analysis. This approach enabled the elucidation of the complex features of order frequency and voluminosity. These probabilistic properties provide useful information for order assessments and can be used for production/chicken supply planning and further scheduling of the industry.

Introduction

For chicken processing, order frequency and voluminosity play crucial roles in determining the bird weight to be supplied to chicken processors because the probabilities of low/high frequency and low/high voluminosity chicken products (stated in term of parts and sizes) require certain weight ranges of birds to be supplied, as shown in Figure 1. Uncertainty in the supply size is affected by biological variation, seasonality, and random factors connected with weather, pests, and other biological hazards (Li, Kramer, Beulens, & van der Vorst, 2010). Hence, in a flock of chickens, the variability of bird weight within a flock is natural and inevitable. When planning the supply of chickens in certain weight ranges, we must also consider the supply of birds in neighboring ranges (Taube-Netto, 1996) given a wide span of chicken ages from 7 to 35 days that corresponds to a weight range of 0.1–3.0 kg (Figure 1). A number of products have a very tight weight range, whereas other products can be produced by a wider weight range, as shown in Table 1.

Simplistically, the simple task of chicken processing involves producing chickens around a fixed mean weight; however, in reality, because of wide product-range planning, the planned processing of chickens is more complicated.

With respect to a market with uncertainty in order, customers (retail chains) require high reliability, but demand is relatively unpredictable from the point of view of the producer. In several cases, the retailers pass all of the uncertainty in demand to the producers by requiring instant delivery within a very short time, without adequate support in forecasting (van Donk, 2001). However, with forecasting by analyzing point-of-sale and scanning data,

* Corresponding author.
E-mail addresses: pkullap@kku.ac.th (K. Piewthongngam), fecotpa@ku.ac.th (T. Athipanyakul).
Peer review under responsibility of Kasetsart University.
[1] Co-first authors.

https://doi.org/10.1016/j.kjss.2018.05.012

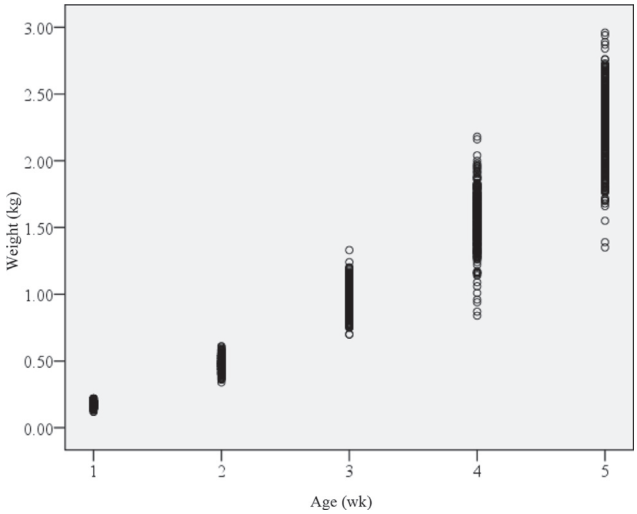

Figure 1 Distribution of chicken weight at age 1−5 weeks

and better cooperation and data sharing between retailer and producer, there are possibilities for a longer lead time. Even with these types of cooperation, the divergent product structures can still result in erratic demand in practice (van Donk, 2001). Poultry processors commonly deal with vast varieties of product (more than 400 products for the studied processor). Hence, forecasting demand on a detailed day-to-day or even weekly level is not easy.

With uncertainties in order frequency and quantity and the weight range of chickens at different ages, the simple task of determining a fixed mean weight of chickens to be supplied is no longer simple. At this point, the problem is stochastic in both demand and supply; this problem needs a tool to facilitate decision-making.

The proper means to investigate the stochastic characteristics of orders is by means of probability theory and the stochastic process method. As stated earlier, orders characterized by several randomly correlated variables (frequency and voluminosity) affect planning and scheduling. Therefore, univariate analysis is unable fully to describe order characteristics and explain the correlations between variables. Hence, a method that addresses multivariate problems is needed. Over the last decade, copulas have emerged as a method to address multivariate problems in several disciplines, such as insurance (Zhao & Zhou, 2010) and climatology (Mirabbasi, Fakheri-Fard, & Dinpashoh, 2012). For example, in the insurance industry, copula has been utilized to estimate the incurred but not reported (IBNR) claim that characterizes the two correlated variables of claim sizes and delays. Correctly predicting their liabilities facilitates insurers in the estimation of loss reserves for IBNR claims (Zhao & Zhou, 2010). In another example of copula application in climatology, Mirabbasi et al. (2012) used copula to characterized the two major drought characteristics, duration and severity, to assess the risk of drought and to prevent water level decline, increased salinity, and accurate water resources management.

To the best of our knowledge, copulas have not yet been applied in the bivariate frequency analysis of order. In this study, several families of two-dimensional copulas were applied to develop a joint order frequency and voluminosity distribution using daily orders obtained from a case study company. The best-fitted copula was selected based on an error analysis and the tail dependence coefficient. These copulas provide useful information for order assessments and can be used for production planning and further scheduling, particularly for determining the most resilient chicken supply weight range for the demand uncertainty.

Materials and Methods

Copula Model

A copular function is a multivariate distribution function that links joint probability distributions to their one-dimensional marginal distributions (Nelsen, 2006). For a bivariate case, according to Sklar (1959), two random variables X and Y with $f_X(x)$ and $f_Y(y)$ are the density functions that correspond to the marginal distribution of $F_X(x)$ and $F_Y(y)$; later, $F_X(x)$ and $F_Y(y)$ are defined as u and v, respectively, in copula form. A copula, C, for this bivariate case, combines these two marginal distributions to provide the joint distribution, $F_{X,Y}(x,y)$, as $F_{X,Y}(x,y) = C(F_X(x), F_Y(y))$ For a more detailed description of a copula function, refer to Nelsen (2006).

For many bivariate distributions, the copula form is the easiest means to express and generate the joint probabilities. This form allows a separate description of the individual marginal distributions and their joint distribution. Copulas also work in the multivariate context; however, this paper will primarily examine bivariate copulas, particularly those defined by a single parameter.

A suitable copula can be determined by selecting the best fitted parameter. In this study, a parametric estimation procedure known as the inference function for margin (IFM) was used to estimate the copula dependence parameter because of its greater simplicity and efficiency

Table 1
Chicken weight range requirement for each product

Part	Size range
Boneless Breast (BB)	1.65–2.40
	1.65–2.80
	1.65–2.85
	1.80–2.20
	2.00–2.80
	2.20–2.75
	2.40–2.75
	2.40–2.80
	2.50–2.70
	Unspecified
Bone in Leg (BIL)	1.60–1.95
	1.90–2.15
	2.05–2.45
	2.20–2.70
	2.40–2.70
	2.50–2.70
	Unspecified
Fillet (FLT)	1.65–2.45
	1.75–2.50
	2.15–2.70
	2.50–2.70
Wing	2.50–2.70

(Mirabbasi et al., 2012). In this study, the bivariates considered were frequency and volume of order.

After obtaining copula parameters, the next step is to select the suitable copula from the families under consideration. For comparison purposes, empirical copulas need to be obtained that are rank-based, empirically joint, cumulative probability measures (Nelsen, 2006). For the bivariate case, the empirical copula of the observed data (u_i, v_i) is

$$C_e(u_i, v_i) = \frac{1}{n}\sum_{i=1}^{n} I\left(\frac{Freq_i}{n+1} \le u_i, \frac{Vol_i}{n+1} \le v_i\right)$$

where n is the sample size; $I(A)$ denotes the indicator variable of the logical expression A and assumes a value of 0 if A is false and 1 if A is true. Additionally, the ranks of the ith observed order frequency and quantity data are represented as $Freq_i$ and Vol_i, respectively (Mirabbasi et al., 2012).

The fitted copula is assessed by comparing the empirical copula $C_e(u_i, v_i)$, with the fitted parametric copula $C_p(u_i, v_i)$ where both are evaluated using the observed data. For other criteria, because a maximum likelihood approach is used to obtain copula parameters, the maximum of the log likelihood values of all tested families may be used as one of the criteria. Additionally, the test of the criteria used the root mean square error (*RMSE*), the Akaike information criterion (*AIC*), the Baysian information criteria (*BIC*), and the Nash-Sutcliffe equation (*NSE*) to select the best-fitted copula function. The model is more efficient when the *RMSE* is close to zero and the *NSE* is close to one. The best-fitted model is determined from the minimum *RMSE*, *AIC*, and *BIC* and the maximum of the log-likelihood and *NSE*.

In this section, the data obtained from a case study company was used. Our case study involved poultry processors and chickens, size, and products. One year of orders for 2011 was obtained. The frequency was represented by the number of orders per week and voluminosity was represented by the total order volume per week. We calculated various basic descriptive statistics of frequency and voluminosity (mean, maximum, minimum, standard deviation, variance, skewness, and kurtosis), as shown in Table 2. The demand frequency and voluminosity of the poultry varied for each chicken part and size. Certain products of 'Boneless breast' (sizes 1.65–2.4 kg, 1.65–2.8 kg, 1.8–2.2 kg) appeared to have demand certainty, whereas a number (size 2.5–2.7 kg) were ordered infrequently, and others ('Boneless breast' size 2.4–2.75 kg) were highly uncertain in terms of both frequency and quantity of order. Certain 'Bone in leg' products (1.6–1.95 kg) solely varied in order quantity but had consistent order frequency, whereas others ('Bone in leg' 2.2–2.7 kg) had highly consistent order quantity but varied in the frequency per week. Hence, it was not easy to forecast on a detailed day-to-day or weekly level.

Results and Discussion

Five families of copulas—Clayton, Gumbel, Frank, Gaussian, and t—were selected to assess their fitness to the observed order data. The parameters of these copulas are shown in Table 3, which indicates the goodness-of-fit of the

Table 2
Descriptive statistic of frequency and voluminosity of poultry parts and sizes in 2011

Part	Size range (kg)	Frequency							Voluminosity						
		Min	Max	Mean	SD	Var	Skew	Kurtosis	Min	Max	Mean	SD	Var	Skew	Kurtosis
Boneless Breast (BB)	1.65−2.40	0	3	0.65	0.79	0.62	0.91	−0.10	0	36.65	8.20	10.75	115.63	1.04	−0.09
	1.65−2.80	0	2	0.71	0.64	0.41	0.31	−0.77	0	42.60	15.54	14.25	203.14	0.31	−1.07
	1.65−2.85	0	1	0.17	0.38	0.41	1.68	0.84	0	58.91	3.64	12.73	203.14	3.55	11.24
	1.80−2.20	0	2	0.29	0.54	0.29	1.63	1.71	0	44.00	5.55	10.47	109.64	1.68	1.96
	2.00−2.80	0	5	0.96	1.10	1.21	1.20	1.57	0	105.17	19.91	22.94	526.17	1.23	1.76
	2.20−2.75	0	4	1.06	0.92	0.84	0.64	0.28	0	67.50	19.36	18.29	334.42	0.78	−0.08
	2.40−2.75	0	4	1.00	1.07	1.14	1.05	0.54	0	69.21	17.48	19.28	371.85	0.96	0.02
	2.40−2.80	0	3	0.25	0.62	0.39	2.66	7.00	0	38.65	3.40	8.04	64.63	2.37	5.57
	2.50−2.70	0	14	3.58	3.68	13.54	0.91	−0.25	0	335.64	69.21	81.19	6592.08	1.23	0.72
	Unspecified	0	2	0.31	0.58	0.33	1.66	1.67	0	44.69	4.72	10.21	104.29	2.05	3.40
Bone in Leg (BIL)	1.60−1.95	0	4	0.42	0.85	0.72	2.32	5.49	0	33.89	2.99	6.91	47.76	2.76	7.56
	1.90−2.15	0	7	1.33	1.84	3.40	1.13	0.20	0	37.50	5.88	9.15	83.81	1.59	1.75
	2.05−2.45	0	3	0.87	0.91	0.82	0.57	−0.90	0	133.73	24.46	34.36	1180.77	1.64	2.06
	2.20−2.70	0	3	1.08	0.99	0.97	0.33	−1.15	0	111.28	22.54	29.97	898.49	1.70	1.99
	2.40−2.70	0	13	1.71	2.37	5.62	2.22	7.44	0	48.89	10.48	13.11	171.90	0.94	−0.14
	2.50−2.70	0	2	0.19	0.44	0.20	2.15	3.97	0	64.08	4.09	11.05	122.08	3.55	14.75
	Unspecified	0	10	4.31	2.89	8.37	0.26	−1.13	0	124.85	28.42	33.07	1093.73	1.72	2.10
Fillet (FLT)	1.65−2.45	0	2	0.29	0.57	0.33	1.78	2.10	0	26.50	3.14	6.38	40.73	1.92	2.82
	1.75−2.50	0	2	0.71	0.64	0.41	0.31	−0.77	0	42.60	15.54	14.25	203.14	0.31	−1.07
	2.15−2.70	0	2	0.25	0.52	0.27	1.91	2.76	0	23.27	2.66	5.87	34.44	2.11	3.26
	2.50−2.70	0	5	0.69	1.13	1.28	1.65	2.44	0	49.30	6.49	12.02	144.53	1.91	2.94
Wing	2.50−2.70	0	1	0.17	0.38	0.15	1.68	0.84	0	58.91	3.64	12.73	161.94	3.55	11.24

best-fitted copula of a primarily Clayton copula and a *t* copula for Boneless breast sizes 1.65−2.8 kg. Because the frequency and voluminosity are suited to characterization by a strong lower tail dependence and nearly upper tail independence, the Clayton is consistent with the results of the correlation and the tail dependence tests stated earlier. Consequently, it was chosen. Another test was to plot the theoretical estimation obtained using the IFM procedure of the Clayton copula for respective chicken parts against the empirical copula for the studied order data. The relation was close to the 45° line and confirmed the use of the copula to characterize the dependence structure and to construct the bivariate model (Mirabbasi et al., 2012).

Copula Applications in Poultry Processing

Generating Dataset from Estimated Copula

The approach to generate the dataset from the copulas used in this paper followed the step suggested by Moazami, Golian, Kavianpour, and Hong (2014). The details of our approach are given as follows:

Let C^2 be the bivariate copula (based on the obtained order frequency and voluminosity) and $H{\equiv}(H_1, H_2)$, where H_1, H_2 are the marginal distributions. To obtain a simulated variable $x{\equiv}(x_1, x_2)$ with marginal distributions of H_1, H_2, the following three steps are required: 1) Estimate the parameter of the copula C^2; 2) Generate two independent uniform variates $u(u_1, u_2)$ using copula C^2 to represent the marginal of H_1, H_2; and 3) Transform the univariate marginal of H_1, H_2 using Sklar's theorem (Sklar, 1959): $x_n = H_n^{-1}(u_n)$. It should be noted that $n = 2$ in this study and also that H_1, H_2 do not necessarily need to belong to the same distribution family. As mentioned previously, as the copula was invariant to the monotonic transformations of the variables, the simulated random variables had the same spatial dependence structures as that of the input data. This is one of the main advantages of copulas in the simulation of spatially dependent random variables. Using this generated dataset, planners are equipped with more advanced operation planning models to explore different scenarios based on these estimated distributions.

Exploring the Effects of Order Patterns on Planning, Marketing, and Product Development Strategies

Let us explore the effects of the joint probabilities of order frequency and voluminosity further on chicken production by assuming a set of demands that are randomly drawn using the method stated in Moazami et al. (2014) and generated prices, from U (130, 210), in Table 4. Each product requires a certain bird weight range, as indicated in Table 1 to be processed. Some of the products have a very tight weight range, whereas other products can be produced in a wider weight range as shown. The task of the poultry processor is to produce chickens around a fixed mean weight; this needs to consider a guideline policy for average chicken size and also the order frequency and voluminosity as well as a distribution weight that corresponds to the average weight of a chicken. If we were to raise an average weight (μ) of 1.8, 1.9,…, 2.8 flocks, that distribute $N(\mu, 0.5)$ with the size of 500,000 birds, to satisfy this assumed set of demands, the cost of using different bird size flocks will be different. The other information used in the demonstration case is the standard carcass and costs. The standard carcass composition of each chicken size used was 25 percent of 'Bone in leg', 19 percent 'Boneless breast', 7.9 percent 'wing', and 3.7 percent 'fillet'. Using this standard carcass, a chicken of size 2.3 kg is composed of 0.58 kg of 'Bone in leg', 0.44 kg of 'Boneless breast', 0.18 kg of 'wing', and 0.09 kg of 'fillet'. The cost of chicken is assumed to be THB 40 per kg (THB 1 is approximately USD 0.03) and the costs of each chicken part are derived from the standard carcass. Shortage and holding

Table 3
Parameter values, the log-likelihood, Akaike information criterion (AIC), Baysian information criteria (BIC), root mean square error (RMSE), and Nash −Sutcliffe equation (NSE) for copula distributions

Part	Size range	Copula	Parameter	Loglikelihood	AIC	BIC	RMSE	NSE
Boneless Breast (BB)	1.65−2.40	Gaussian	0.850	625.936	−1249.872	−1244.964	0.114	0.816
		T	0.877,1.000	937.143	−1872.286	−1867.378	0.051	0.959
		Clayton	12.901	**1723.102**	**−3444.204**	**−3439.296**	**0.001**	0.980
		Gumbel	5.370	1307.342	−2612.684	−2607.776	0.024	**0.989**
		Frank	21.338	1211.528	−2421.056	−2416.148	0.056	0.951
	1.65−2.80	Gaussian	0.896	790.505	−1579.010	−1574.102	0.113	0.774
		T	0.979, 1.568	**1774.258**	**−3546.516**	**−3541.608**	0.041	0.971
		Clayton	8.506	1353.663	−2705.326	−2700.418	**0.003**	0.930
		Gumbel	5.268	1254.816	−2507.632	−2502.724	0.062	0.923
		Frank	22.178	1187.905	−2373.810	−2368.902	0.029	**0.984**
	1.65−2.85	Gaussian	0.908	882.841	−1763.683	−1758.775	0.038	**0.975**
		T	0.931	1252.401	−2502.802	−2497.894	0.053	0.940
		Clayton	8.645	**1388.274**	**−2774.548**	**−2769.640**	**0.003**	0.930
		Gumbel	4.098	1028.620	−2055.240	−2050.332	0.056	0.946
		Frank	17.444	1093.217	−2184.434	−2179.526	0.055	0.925
	1.80−2.20	Gaussian	0.902	817.381	−1632.762	−1627.854	0.051	0.950
		T	0.980,1.000	1874.367	−3746.734	−3741.826	0.082	0.833
		Clayton	37.279	**2655.750**	**−5309.500**	**−5304.592**	**0.003**	0.945
		Gumbel	9.413	1835.531	−3669.062	−3664.154	0.036	**0.976**
		Frank	56.166	2046.799	−4091.598	−4086.690	0.059	0.947
	2.00−2.80	Gaussian	0.975	1460.268	−2918.536	−2913.628	0.076	0.888
		T	0.998,1.000	1874.367	−3746.734	−3741.826	0.081	0.898
		Clayton	33.481	**2452.577**	**−4903.154**	**−4898.246**	**0.004**	0.913
		Gumbel	10.250	1907.914	−3813.828	−3808.920	0.041	**0.971**
		Frank	50.665	2023.870	−4045.740	−4040.832	0.095	0.842
	2.20−2.75	Gaussian	0.883	785.718	−1569.436	−1564.528	0.094	0.847
		T	0.927,1.081	1356.922	−2711.844	−2706.936	0.042	0.962
		Clayton	9.625	**1501.680**	**−3001.360**	**−2996.452**	**0.004**	0.924
		Gumbel	3.607	874.249	−1746.497	−1741.589	0.039	**0.969**
		Frank	17.133	1042.669	−2083.338	−2078.430	0.038	0.968
	2.40−2.75	Gaussian	0.946	1166.432	−2330.864	−2325.956	0.095	0.818
		T	0.944,1.000	1343.257	−2684.514	−2679.606	0.065	0.923
		Clayton	12.027	**1665.791**	**−3329.582**	**−3324.674**	**0.001**	**0.986**
		Gumbel	4.879	1191.822	−2381.644	−2376.736	0.036	0.978
		Frank	21.804	1189.662	−2377.324	−2372.416	0.032	0.981
	2.40−2.80	Gaussian	0.959	1263.271	−2524.542	−2519.634	0.083	0.867
		T	0.958,1.000	1476.340	−2950.680	−2945.772	0.039	0.971
		Clayton	56.493	**3010.728**	**−6019.456**	**−6014.548**	**0.003**	0.960
		Gumbel	16.798	2437.804	−4873.608	−4868.700	0.038	0.974
		Frank	80.885	2386.988	−4771.976	−4767.068	0.037	**0.979**
	2.50−2.70	Gaussian	0.959	1311.819	−2621.638	−2616.730	0.051	0.948
		T	0.963,2.027	1381.883	−2761.766	−2756.858	0.039	**0.972**
		Clayton	11.634	**1611.583**	**−3221.166**	**−3216.258**	**0.002**	0.941
		Gumbel	5.594	1306.783	−2611.566	−2606.658	0.146	0.328
		Frank	26.744	1423.694	−2845.388	−2840.480	0.050	0.957
	Unspecified	Gaussian	0.8407	648.724	−1295.447	−1290.539	0.043	0.967
		T	0.902,1.000	1068.043	−2134.086	−2129.178	0.043	**0.968**
		Clayton	15.567	**1857.863**	**−3713.726**	**−3708.818**	**0.004**	0.920
		Gumbel	6.978	1549.640	−3097.280	−3092.372	0.082	0.880
		Frank	28.983	1527.275	−3052.550	−3047.642	0.046	0.959
part	size range	Copula	parameter	Loglikelihood	AIC	BIC	RMSE	NSE
BL (Bone in Leg)	1.60−1.95	Gaussian	0.935	1068.730	−2135.460	−2130.552	0.044	0.966
		T	0.8913, 1.000	1066.940	−2131.880	−2126.972	0.037	0.976
		Clayton	24.799	**2373.371**	**−4744.742**	**−4739.834**	**0.001**	**0.980**
		Gumbel	11.221	2011.345	−4020.690	−4015.782	0.059	0.901
		Frank	42.747	1812.936	−3623.872	−3618.964	0.035	0.972
	1.90−2.15	Gaussian	0.830	565.222	−1128.444	−1123.536	0.085	0.855
		T	0.869,1.000	963.767	−1925.535	−1920.627	0.059	0.922
		Clayton	12.944	**1709.920**	**−3417.840**	**−3412.932**	**0.001**	**0.982**
		Gumbel	5.733	1337.944	−2673.888	−2668.980	0.041	0.973
		Frank	23.042	1272.393	−2542.786	−2537.878	0.073	0.905
	2.05−2.45	Gaussian	0.931	1089.741	−2177.482	−2172.574	0.137	0.714
		T	0.971,1.000	1726.793	−3451.586	−3446.678	0.074	0.893
		Clayton	19.090	**2127.413**	**−4252.826**	**−4247.918**	**0.003**	0.950
		Gumbel	4.472	1081.335	−2160.670	−2155.762	0.123	0.566
		Frank	29.379	1495.383	−2988.766	−2983.858	0.047	**0.953**

(*continued on next page*)

Table 3 (*continued*)

part	size range	Copula	parameter	Loglikelihood	AIC	BIC	RMSE	NSE
	2.20–2.70	Gaussian	0.846	622.363	−1242.725	−1237.817	0.042	0.969
		T	0.923,1.000	1187.187	−2372.374	−2367.466	0.089	0.808
		Clayton	9.140	**1326.230**	**−2650.460**	**−2645.552**	**0.002**	0.961
		Gumbel	3.029	735.706	−1469.412	−1464.505	0.087	0.823
		Frank	14.238	929.539	−1857.078	−1852.170	0.035	**0.976**
	2.50–2.70	Gaussian	0.943	1114.377	−2226.754	−2221.846	0.030	**0.983**
		T	0.914,1.000	1357.579	−2713.158	−2708.250	0.038	0.964
		Clayton	24.031	**2273.364**	**−4544.728**	**−4539.820**	**0.001**	0.976
		Gumbel	11.010	1938.771	−3875.542	−3870.634	0.046	0.961
		Frank	42.603	1795.570	−3589.140	−3584.232	0.043	0.968
	Any size	Gaussian	0.652	282.081	−562.163	−557.255	0.033	**0.977**
		T	0.700,3.440	391.386	−780.771	−775.863	0.048	0.961
		Clayton	2.811	**602.192**	**−1202.384**	**−1197.476**	**0.002**	0.931
		Gumbel	1.838	313.581	−625.161	−620.253	0.078	0.902
		Frank	6.384	370.042	−738.084	−733.176	0.044	0.954
FLT (Fillet)	1.65–2.45	Gaussian	0.932	997.783	−1993.565	−1988.657	0.036	0.974
		T	0.9342,1.000	1207.606	−2413.212	−2408.304	0.070	0.891
		Clayton	31.087	**2511.656**	**−5021.312**	**−5016.404**	**0.004**	0.891
		Gumbel	9.025	1730.709	−3459.418	−3454.510	0.031	**0.982**
		Frank	48.003	1921.863	−3841.726	−3836.818	0.055	0.957
	1.75–2.50	Gaussian	0.945	1091.501	−2181.002	−2176.094	0.104	0.768
		T	0.887	1012.123	−2022.246	−2017.338	0.032	**0.979**
		Clayton	57.302	**3021.461**	**−6040.922**	**−6036.014**	**0.003**	0.937
		Gumbel	15.169	2347.693	−4693.386	−4688.478	0.050	0.965
		Frank	83.851	2494.579	−4987.158	−4982.250	0.043	0.969
	2.15–2.70	Gaussian	0.934	1002.670	−2003.340	−1998.432	0.033	0.977
		T	0.916,1.000	1127.110	−2252.220	−2247.312	0.030	**0.982**
		Clayton	32.958	**2569.338**	**−5136.676**	**−5131.768**	**0.006**	0.903
		Gumbel	9.604	1833.077	−3664.154	−3659.246	0.076	0.908
		Frank	48.868	2009.054	−4016.108	−4011.200	0.074	0.866
	2.50–2.70	Gaussian	0.921	909.948	−1817.896	−1812.988	0.036	0.968
		T	0.936,1.000	1308.245	−2614.490	−2609.582	0.036	0.976
		Clayton	22.5081	**2238.303**	**−4474.606**	**−4469.698**	**0.005**	0.904
		Gumbel	9.860	1937.437	−3872.874	−3867.966	0.030	**0.983**
		Frank	38.475	1702.113	−3402.226	−3397.318	0.059	0.934
WING	2.50–2.70	Gaussian	0.638	256.506	−511.012	−506.104	0.052	0.956
		T	0.576,1.000	335.249	−668.497	−663.589	0.068	0.912
		Clayton	8.898	**1424.596**	**−2847.192**	**−2842.284**	**0.001**	**0.987**
		Gumbel	3.801	991.380	−1980.759	−1975.851	0.055	0.913
		Frank	16.511	969.489	−1936.978	−1932.070	0.035	0.972

Note: The bold figures indicate the best-fitted copula using respective the goodness-of-fit tests

Table 4
Demand set generated from respective copulas and prices from rounding numbers of *U* (130, 210)

Part	Chicken weight range (kg)	Demand (kg)	Price (THB/kg)
Boneless breast (BB)	1.65–2.40	27,648	156
	1.65–2.80	41,958	190
	1.65–2.85	21,849	172
	1.80–2.20	3217	183
	2.00–2.80	47,735	191
	2.20–2.75	12,581	185
	2.40–2.75	58,437	151
	2.40–2.80	21,120	210
	2.50–2.70	37,040	149
	Unspecified	15,158	144
Bone in leg (BIL)	1.60–1.95	12,906	170
	1.90–2.15	2142	159
	2.05–2.45	49	181
	2.20–2.70	17,842	186
	2.50–2.70	13,500	154
	Unspecified	9885	163
Fillet (FLT)	1.65–2.45	16,661	162
	1.75–2.50	12,580	184
	2.15–2.70	10,094	182
	2.50–2.70	15,097	133
Wing	2.50–2.70	25,266	133

costs are assumed to be THB 25 per kg and THB 10 per kg, respectively. To set a production plan, we apply the following concept. Because each product requires a different weight range, the weight that is used is the one with the lowest cost. In the case of insufficient supply, the next lowest-cost size is chosen.

The intuitive implications of the model are: 1) Regarding the supply size, by assuming the flock size of 500,000 birds that is assumed to be distributed normally, $N(\mu, 0.5)$, with average weight (μ) of 1.8, 1.9,…, 2.8 kg. By applying standard carcass, it will automatically result in different levels of chicken parts and sizes. For example, using a flock of 500,000 birds with $\mu = 1.8$ kg average weight with a normal distribution, we will get 19,820 and 17,262 of chicken sizes 1.8 kg and 2.1 kg, respectively, along with other sizes. Chicken parts derived from chicken size 1.8 kg are BB, BIL, FLT and wings of 3,337, 4,391, 650, and 1,388 kg, respectively, while the same size of carcass yields 2,787, 3,667, 543, and 1,159 kg of BB, BIL, FLT and wings, respectively, for a 2.1 kg average weight; 2) Depending on the cost and availability, the production plan stated earlier results in different usages and inventory levels of chicken parts and sizes. For example, for a weight average of 1.8 kg,

Table 5
Profit, revenue and cost resulting from supplying 500,000 chickens with $\sigma = 0.5$ for flocks of 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8 kg average weight

	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8
Profit	11,110,123	12,265,353	13,217,951	13,849,846	**14,100,557**	13,711,565	12,914,226	11,779,931	10,295,377	8,501,297	6,153,776
Revenue	38,320,150	41,339,189	44,075,806	46,278,076	47,802,670	**48,481,367**	48,288,398	47,301,563	45,436,038	42,784,961	39,091,691
Cost	**27,210,027**	29,073,836	30,857,855	32,428,230	33,702,113	34,769,802	35,374,172	35,521,632	35,140,661	34,283,664	32,937,915

Note: The bold figures indicate optimum number of the profit, revenue, and cost

the usages of BB, BIL, FLT and wings from carcass size of 1.8 kg are 3,337, 0, 650, and 0 kg, respectively, and hence, the inventory levels are 0, 4391, 0, and 1,388 kg respectively. Likewise, for the average weight of 2.1, the usages from a carcass size of 1.8 kg are 2,787, 0, 543, and 0, and hence, the inventory levels are 0, 3,667, 0, and 1,159, respectively. This implies the importance of the size of chicken supply when using this concept. Depending on the order pattern, the size of chicken supply should be determined accordingly; 3) Notice that this particular order pattern and chicken supply lead to inventory levels building up for wings and parts of large-sized chickens. Furthermore, after the killing process, scalding, defeathering, giblet processing, and chilling and maturation, birds are cut-up and processed further following the processing plan and orders. The parts that are yet to be processed are frozen and kept in cold storage as inventory. Normally, this will result in strategies such as expanding the market or launching a product development program for high inventory chicken parts and determining the proper size of chicken to supply can be executed. The processor needs to communicate the needed chicken size to the farmers. This information facilitates communication between processors and farms and, in turn, lowers the cost of the chain (feed cost, inventory cost, production cost). Those who raise larger-sized chickens should be farmers who have the feed costs of excess chicken weight and the opportunity costs of raising a new batch of chicken; 4) A shortage of some chicken parts/sizes may occur at different levels for each average weight of the flock. This particular set of demand and supply requirements depicts a shortage for Fillet sizes 1.65–2.45 and 1.75–2.50 when supplying a flock with an average weight of 2.5–2.8 kg, while a shortage of Fillet size 2.50–2.70 results when supplying a flock with an average weight of 1.8–2.1 kg. Furthermore, there is no shortage of any product when supplying a flock with an average weight of 2.3–2.4 kg. Knowing this information in advance, the processor can prepare for strategic outsourcing to smooth out the shortage while satisfying the demand.

As demonstrated in Table 5, the cost of using an average size of 1.8 kg is around THB 27 million, whereas it is around THB 32 million when using a 2.8 kg flock. For this studied set of demand and supply figures, the average size of 2.3 kg gives the highest revenue while the average size of 2.2 kg gives the highest profit. To explore further, the simulation was repeated with 1,000 sets of demand to obtain the profit of each flock size. It was found that the average weight of 2.1 kg resulted in the highest maximum and the least minimum profit, while the average weight of 2.2 kg produced the highest mean. However, supplying the optimal average bird size would require more advanced operational planning and a more sophisticated mathematical model and/or more involved statistical computations. Finally, expressing order frequency and voluminosity in a copula function assist planners in the utilization of advanced modeling and operational planning.

Taube-Netto (1996) stated similar benefits as well as other benefits that are likely to occur: a reduction of the feed consumption ratio, higher value products, and avoiding loss on income due to the profile of the birds conflicts with market needs. The feed consumption ratio is defined as the quotient of the total feed consumption and the weight of a bird. The desirable ratio is as small as possible. The main contribution of a less-than-optimum feed consumption ratio is raising the birds to an unsuitable age. For example, deciding to postpone slaughtering a flock at a certain age and even leaving it in the field for one more day, implies raising the average weight of the flock at the expense of higher feed consumption than that observed until then. The benefits of higher value products and avoiding loss on income due to the profile of birds conflicting with market needs are two sides of the same coin. When both the proper bird and flock size are achieved, the size with less demand is automatically reduced. Being able to quantify the unwanted sizes equips the processor with the best possible way to market them in advance.

To implement this concept, the processor planning team should estimate demand for families of products over 12–18 months in order to characterize the products by the range of bird weights, including carcass weight and parts weight, from which it can be produced. Then, the bird size policy according to estimated demand can be determined. However, the average bird size and flock size can be adjusted according to the actual orders in the following month. Then, for the slaughtering schedule, an actual weight distribution of each farm and confirmed demand can be synchronized to determine the slaughtering schedule for each farm for the next 7–15 days.

Conclusions

Reliable estimates of orders are essential for the processing plan because ordering uncertainties in the major input data can propagate into the operation planning model. Furthermore, detailed ordering information and a better understanding of its distributions are important for the determination of the chicken size supply, production plan, inventory and marketing management, and new product development. In this study, a copula function was used to explain the joint distribution between order frequency and voluminosity. High volatility in certain parts/sizes of chicken will affect inventory management because of the nature of the co-products of a chicken carcass. We demonstrated the effects of order joint probability on the chicken size supply policy and other operations. An order pattern requires a certain size of supply. Improper size is costly for the firm and its chain members. Reliable estimates of orders enable a firm to react to these uncertainties in advance. Hence, this information enables better intra/inter supply chain communication. Well-utilized information would likely save the firm and its farmers from high inventory and production costs and hence lead to more profitability. In particular, expressing order frequency and voluminosity in a copula function facilitates more advanced operational planning, which requires a sophisticated mathematical model and/or statistical works.

Conflict of interest

There is no conflict of interest.

Acknowledgments

This research was funded by the National Research University of Khon Kaen University Fund under project FC 6.1.5 Ph.D. and Betagro Science Center Company Limited.

References

Li, Y., Kramer, M. R., Beulens, A. J. M., & van der Vorst, J. G. A. J. (2010). A framework for early warning and proactive control systems in food supply chain networks. *Computers in Industry, 61*(9), 852–862.

Mirabbasi, R., Fakheri-Fard, A., & Dinpashoh, Y. (2012). Bivariate drought frequency analysis using the copula method. *Theoretical and Applied Climatology, 108*(1), 191–206.

Moazami, S., Golian, S., Kavianpour, M. R., & Hong, Y. (2014). Uncertainty analysis of bias from satellite rainfall estimates using copula method. *Atmospheric Research, 137*, 145–166.

Nelsen, R. B. (2006). *An introduction to Copulas*. New York, NY: Springer.

Sklar, A. (1959). In *Fonctions de Ŕepartition à n Dimensions et Leurs Marges* (Vol. 8, pp. 229–231). Paris, France: Publications de ĬInstitut de Statistique de ĽUniversiť̌e de. [in French]

Taube-Netto, M. (1996). Integrated planning for poultry production at Sadia. *Interfaces, 26*(1), 38–53.

van Donk, D. P. (2001). Make to stock or make to order: The decoupling point in the food processing industries. *International Journal of Production Economics, 69*, 297–306.

Zhao, X.-B., & Zhou, X. (2010). Applying copula models to individual claim loss reserving methods. *Insurance Mathematics and Economics, 46*(2), 290–299.