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A non-linear program to find an approximate location of a second warehouse: A case study

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ABSTRACT

A mathematical model was developed to estimate the location of a second warehouse for a case study in Bangkok. A non-linear program was developed based on the Load Distance Technique. The objective function was to minimize the sum of weighted straight-line distances from the first or second warehouse to either vendors or customers. The straight-line distance was determined using the principle of Pythagorean triples and was weighted by the shipment frequency and the shipment cost rate. The model was then solved using the Microsoft Excel Solver upgraded to the Premium Solver Platform. The starting solutions were randomly set within a specified range to obtain different local optimal solutions. The best one was finally selected to be the approximate location of the second warehouse. Sensitivity to customer demands was conducted and some useful recommendations are provided.

Introduction

This paper extends the study of Monthatipkul (2012). That author determined a location of a second warehouse for a case study (a paper wholesaler in Bangkok). The goal was to decrease the total transportation cost in the long run. A non-linear model was first formulated and then solved using the Premium Solver Platform V11. By diversifying initial solutions, the author obtained the best local optimal solution representing the approximate location of the second warehouse. Even though the author ran some simulations and concluded that opening the second warehouse using the proposed solution could save transportation costs in the long run compared to using only the first warehouse to supply all customers, that research ignored the transportation costs from the supply side. In reality, transport costs from suppliers have significant effects on the selection of a facility's location. A site far away from suppliers invariably results in high transportation costs. Using the same case, this paper strengthens the model proposed in Monthatipkul (2012) to cover the supply side issue.

The problem description explained in Monthatipkul (2012) can be re-summarized as follows. The case study purchases products from many suppliers and distributes them to customers using various types of trucks. The head office is located in the middle of Bangkok, but its warehouse is in southern Bangkok at latitude 13.65841 and longitude 100.47126 (N13.65841, E100.47126). There are almost 1,000 customers (printing, retailers, copier centers, and companies, among others) whose sites are dispersed around Bangkok and the city perimeter. The last three years of records indicate that the sales volume has reached roughly 50 million kilograms of paper. The biggest customer from the west buys 1.7 million kilograms, while the smallest customer in the middle of Bangkok buys only

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45 thousand kilograms. Daily, the case study receives customer orders by telephone and has a cutoff time of 4 p.m. All pre-loading jobs must be performed and the truck schedules/routes must be prepared by 7 p.m. The trucks are loaded the next morning and depart from the warehouse before 8:30 a.m., making a round trip with from 5 to 15 destinations before returning to the warehouse. A second round trip seldom happens.

The main concern of the case study was to reduce the total distribution cost in the long run. Despite determining suitable daily routes by considering related costs and truck capacities, high transportation costs are a major problem due to the inappropriate location of the current warehouse. One possible solution to the problem selected by top management was to open a new warehouse (the second warehouse), in the most suitable location. Some customers would be served by the new warehouse, while the remaining customers would still receive deliveries from the old one. Thus, the main questions in this research were where to locate the second warehouse and how to allocate customers to each warehouse, so that the total transportation cost in the long run was reduced.

To simplify the analysis without loss of generality, the following assumptions were made:

1) Geographical coordinates are used for location identification. They were sourced from the company database (for example, the most significant customer is at N13.77165, E100.39584). However, due to confidentiality requirements, the coordinates of some major customers have been modified for this exercise.

2) The total transportation cost in the long run depends on the actual distances. However, actual determination is cumbersome because of unpredictable factors such as future road construction and traffic conditions, among others. Thus, this research used the straight-line distance instead because this research focused on long-term planning. The result was to used to indicate an approximate location of the new warehouse, with a real-world survey being conducted prior to implementation.

3) The straight-line distance was calculated using Equation (1):

$$\text{Straight} - \text{line distance} = \left\{ (\text{diff\_N})^2 + (\text{diff\_E})^2 \right\}^{1/2} \quad (1)$$

where, diff_N and diff_E are the differences between the latitudes/longitudes of any two points.

4) The straight-line distance was weighted by the shipment frequency and the shipment cost rate. The shipment frequency equals the expected overall demand divided by the shipment size. The shipment cost rate is a multiple of fuel price (baht/liter) and the average fuel consumption rate of vehicles (liter/kilometer).

5) The expected overall demand of each customer was forecast by experts from the marketing department. They used historical data and other necessary parameters from marketing plans and economic growth estimates. The forecast was required to cover a particular period, such as three years.

6) All expected customer demands must be fulfilled by one of the two warehouses, which in turn, are supplied by company vendors. The material balance principle must apply and no shortages are allowed. A customer cannot be assigned to both warehouses simultaneously.

7) Despite many types of papers being traded in practice, this research assumed a single sales unit (kilogram).

8) This paper considered various types of trucks—ten-wheels, six-wheels, and four-wheels. Their fuel consumption rates varied; however, their capacities were ignored.

9) Each vendor can supply every type of paper and has sufficient capacity. Both warehouses must be supplied by at least one vendor.

The remainder of the paper is organized as follows. The next section contains a literature review. Section A Mathematical Model proposes a model for estimating a location of the second warehouse. A solution approach and an illustrative example are then given in the subsequent section. The next section provides an application of the proposed model and some sensitivity analysis. The last section gives a summary.

Literature Review

The facility location problem mainly concerns selecting or placing facilities to serve customer demands efficiently. The following review concludes a literature survey which mainly focused on the continuous facility location problem.

The continuous facility location problem can be briefly stated as: Given n-dimensional points (vertices) $P_1, P_2, \ldots, P_p$ in $\mathbb{R}^n$ and positive multipliers (weights) $\omega_1, \omega_2, \ldots, \omega_p \in \mathbb{R}_+$, find a point P^* that minimizes $\Sigma_p\, \omega_p\, ||P^* - P_p||$, where $||P^* - P_p||$ denotes the Euclidean norm of $P^* - P_p$.

Study in this field probably started in the 17th century with the so-called Fermat Problem. Fermat (as cited in Vygen, 2004) first proposed a model with $n = 2$, $p = 3$, and $\omega_p = 1$ for all p, which was later solved to determine point P^* by many recognized mathematicians, Torricelli, Cavalieri, Simpson, and Heinen (see Vygen, 2004, p. 3). Weber (1909) extended the Fermat Problem by further considering imbalance of ω_p. The Weber Problem is famous in location theory and involves finding a point in a plane and minimizing the total transportation costs from this point to many destination points with different costs per unit distance. Weiszfeld (1937) proposed an iterative algorithm (Weiszfeld's Algorithm) for the Weber Problem. A more complex problem with $p > 3$ was later studied by Kulin and Kuenne (1962). They outlined an iterative numerical method for the general Weber Problem in spatial economics. Tellier (1972) also presented a direct numerical solution to the Weber Problem. Kulin (1973) gave a proof of convergence of point P* (whether it was an optimum point or not), which was later confirmed by Rautenbach, Struzyna, Szegedy, and Vygen (2004).

A problem considering both attractive and repulsive forces ($\omega_p \leq 0$) from each point was formulated and solved by Tellier (as cited in Tellier, 1985) and further investigated by Chen, Hansen, Jaumard, and Tuy (1992). Bajaj (1988) claimed that the Weber Problem with the line-restriction and three-dimensional version was not solvable by radicals over the field of rationals; solution approaches must apply numerical approximations since there was no exact algorithm based on using arithmetic operations. In addition, the Fermat-Weber Problem could be considered as a weighted median problem (see Korte & Vygen, 2000) if all given points were collinear. This situation formed a concrete concept for solving the location problem separately for each coordinate, which was later called the Center of Gravity method.

An extension of Weiszfeld's algorithm covering a more general cost function was studied by Vardi and Zhang (2001). Struzyna (2004) and Szegedy (2005) also extended Weiszfeld's algorithm to a more general problem, where some given points were already linked to some points in $\mathbb{R}^n$, and the main task was to find a solution for the remaining points in order to minimize the total Euclidean distance. Unfortunately, Weiszfeld's algorithm converged very slowly (Drezner, Klamroth, Schöbel, & Wesolowsky, 2002). Brimberg, Drezner, Mladenović, and Salhi (2014) proposed a local search for solving continuous location problems. The main idea was to shift between the continuous and discrete spaces. It gave very competitive results when tested on the continuous p-median problem. Drezner, Brimberg, Mladenović, and Salhi (2015) presented three heuristic methods for solving the multi-source Weber problem in the plane: a constructive heuristic, a decomposition method, and a new, efficient neighborhood structure.

The current article uses an application of Struzyna (2004) and Szegedy (2005). It contributes a model for finding a new approximate location of the second warehouse in the particular setting of the case study, in which some destination points (customers) will be linked to the new warehouse, while all remaining points (customers) will be linked to the first warehouse.

A Mathematical Model

The following notation is used in the proposed model.
Notation:

i customer index ($i = 1, 2, 3, \ldots, I$)
j vendor index ($j = 1, 2, 3, \ldots, J$)
α_i weight factor related to customer i
β_{1j} weight factor related to vendor j supplying goods to the first warehouse
β_{2j} weight factor related to vendor j supplying goods to the second warehouse
f fuel price (Baht/liter)
CR_i average fuel consumption rate of vehicles sent to customer i (liter/kilometer)
DR_i expected overall demand of customer i for a particular long period (unit)
OR_i average shipment size of customer i (unit)
CV_j average fuel consumption rate of vehicles sent from vendor j (liter/kilometer)
OV_j average shipment size of vendor j (unit)
A_i x-coordinate (longitude) of customer i
B_i y-coordinate (latitude) of customer i
E_j x-coordinate (longitude) of vendor j
F_j y-coordinate (latitude) of vendor j
x_1 x-coordinate (longitude) of the first warehouse (given data)
y_1 y-coordinate (latitude) of the first warehouse (given data)
R_{1i} an imaginary straight line from the first warehouse to customer i
R_{2i} an imaginary straight line from the second warehouse to customer i
V_{1j} an imaginary straight line from the first warehouse to vendor j
V_{2j} an imaginary straight line from the second warehouse to vendor j
M a large positive number
Main decision variables:
x_2 x-coordinate (longitude) of the second warehouse
y_2 y-coordinate (latitude) of the second warehouse
s_{1i} a binary number, if R_{1i} is selected, $s_{1i} = 1$, otherwise $s_{1i} = 0$
s_{2i} a binary number, where $s_{2i} = 1 - s_{1i}$, it is noted that $s_{2i} = 1$ representing R_{2i} is selected
t_{1j} a binary number, if V_{1j} is selected, $t_{1j} = 1$, otherwise $t_{1j} = 0$
t_{2j} a binary number, if V_{2j} is selected, $t_{2j} = 1$, otherwise $t_{2j} = 0$
DV_{1j} expected overall demand placed from the first warehouse to vendor j (unit)
DV_{2j} expected overall demand placed from the second warehouse to vendor j (unit)

The objective function is expressed as Equation (2):

$$\text{minimize } z = \sum_i \alpha_i R_{1i} s_{1i} + \sum_i \alpha_i R_{2i} s_{2i} + \sum_j \beta_{1j} V_{1j} t_{1j} + \sum_j \beta_{2j} V_{2j} t_{2j} \quad (2)$$

subject to

$$\alpha_i = f\, CR_i (DR_i / OR_i) \quad \text{for all } i \quad (3)$$

$$\beta_{1j} = f\, CV_j (DV_{1j} / OV_j) \quad \text{for all } j \quad (4)$$

$$\beta_{2j} = f\, CV_j (DV_{2j} / OV_j) \quad \text{for all } j \quad (5)$$

$$R_{1i} = \left\{ (A_i - x_1)^2 + (B_i - y_1)^2 \right\}^{1/2} \quad \text{for all } i \quad (6)$$

$$R_{2i} = \left\{ (A_i - x_2)^2 + (B_i - y_2)^2 \right\}^{1/2} \quad \text{for all } i \tag{7}$$

$$V_{1j} = \left\{ (E_j - x_1)^2 + (F_j - y_1)^2 \right\}^{1/2} \quad \text{for all } j \tag{8}$$

$$V_{2j} = \left\{ (E_j - x_2)^2 + (F_j - y_2)^2 \right\}^{1/2} \quad \text{for all } j \tag{9}$$

$$s_{2i} = 1 - s_{1i} \quad \text{for all } i \tag{10}$$

$$\sum_i DR_i s_{1i} \le \sum_j DV_{1j} t_{1j} \tag{11}$$

$$\sum_i DR_i s_{2i} \le \sum_j DV_{2j} t_{2j} \tag{12}$$

$$DV_{1j} \le Mt_{1j} \quad \text{for all } j \tag{13}$$

$$DV_{2j} \le Mt_{2j} \quad \text{for all } j \tag{14}$$

$$x_2, y_2, DV_{1j}, DV_{2j} \ge 0 \quad \text{for all } j \tag{15}$$

$$s_{1i}, s_{2i}, t_{1j}, t_{2j} \text{ are binary} \quad \text{for all } i, j \tag{16}$$

The problem can be stated as “Determine the location of the second warehouse (x_2, y_2) so as to minimize the sum of weighted, straight-line distances from the two warehouses to their allocated customers and vendors”. Equation (2) is the sum of those weighted, straight-line distances represented by the terms α_i R_{1i}, β_{1j} V_{1j}, α_i R_{2i}, and β_{2j} V_{2j}. The variables s_{1i} and t_{1j} will be equal to one if customer i or vendor j is assigned to the first warehouse, otherwise, they will be equal to zero. The variables s_{2i} and t_{2j} will be equal to one if customer i or vendor j is assigned to the second warehouse, otherwise, they will be equal to zero.

Equations (3)–(5) express the weights of the straight-line distances as functions of the fuel price, average consumption rates of vehicles, associating demands, and shipment sizes. A higher weight will be assigned to a customer or a vendor whose expected overall demand is high and shipment size is low because it will result in a high frequency of shipment. The higher fuel consumption rates of vehicles and higher prices for their fuel will add to the weighting, too. Equations (6)–(9) estimate the straight-line distances between the warehouses and their connecting customers or vendors using the principle of Pythagorean triples. They are all non-linear equations resulting in the overall model being a non-linear program.

Equation (10) presents a condition if $s_{1i} = 1$, then $s_{2i} = 0$ or vice versa. Each customer must be assigned to only one warehouse. Inequalities 11 and 12 are to ensure that the product quantity distributed via the first or second warehouse must not be greater than the product quantity supplied by its vendors. Inequalities 13 and 14 are the Big-M constraints. The variables DV_{1j} and DV_{2j} must be zero if the binary variables t_{1j} and t_{1j} are zero. If the variables t_{1j} and t_{1j} are equal to one, the variables DV_{1j} and DV_{2j} can be positive numbers. Finally, Equations (15) and (16) are the non-negativity and the binary conditions, respectively.

A Solution Approach and an Illustrative Example

The following numerical example depicts the model as presented earlier. Consider a case involving a single product, four vendors and 20 customers. The first warehouse is at coordinates (2, 3). The number of customers and vendors is within the range 1–10 (see Figure 1). Table 1 gives all data used in the example, which is solved using Microsoft Excel Solver upgraded by the Premium Solver Platform (Options GRG Nonlinear and Evolutionary are selected alternately). The starting solutions are randomly diversified within the range 1–10 in order to obtain different local optimal solutions. After a local optimum is found, the solver is repeatedly run until the current result is no longer improved on for five iterations. The lowest optimal solution is selected to be the final result. Table 2 and Figure 1 show the results.

From Table 2, the second warehouse is at coordinates (2.8, 7.6). The z-value is equal to 19,146.9. The values of s_{1i} and s_{2i} show the allocation decision for customers. It is noted that each vendor or customer is assigned to the nearest warehouse for which its associated straight-line distance is minimal. Figure 1 also shows the movement of products for the solution to this example. The first warehouse is supplied by the vendor at coordinates (2, 2). Products are transferred to seven customers whose sites are on the right-hand side. The second warehouse at location (2.8, 7.6) receives products from a vendor in the upper-right corner, then distributes them to 10 customers whose sites are all in the upper side and also three customers to the right-hand side. The remaining two vendors in the lower-left corner are not allocated to any warehouse in the current solution. Finally, it is noted that the total amount of products sold by each warehouse to its customers is equal to that of products bought from its vendors (see Table 2).

Application to the Case Study

The proposed model was applied to the case study. Numerical experiments were conducted using the Premium Solver Platform (www.solver.com). The experiments were based on real data from the case study with some modifications. There were 910 customers, whose sites are dispersed around Bangkok within N13.52369 to N14.12159 and E100.24878 to E100.85716. The first warehouse is at N13.65841, E100.47126. All necessary given data were extracted from the case study and are summarized in Table 3. Note that DR_i is generated within the range 45,000–2,000,000 based on forecasts from the marketing department.

Table 4, Figures 2 and 3 summarize the results. The second warehouse is located on the northeast side of the first warehouse at N13.80753, E100.56386. The straight-line distance between them is 9.5 km, while the actual distance is 30.1 km (routing by map.longdo.com). From Figure 2, the first warehouse is supplied by its nearest vendor, with the straight-line distance between them being

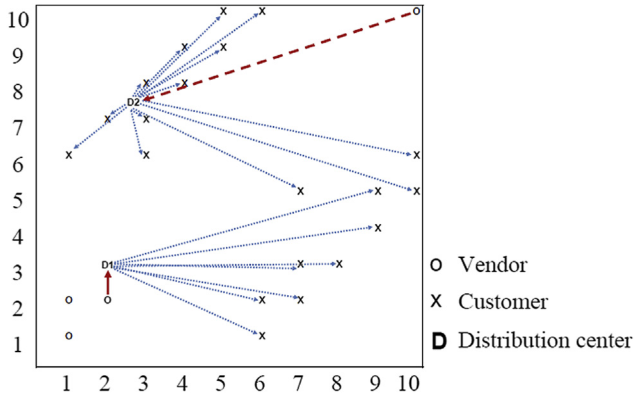

Figure 1 Graphical solution to the numerical example

Table 1
Given parameters used in the example

Vendor j	E_j	F_j	OV_j	CV_j	Other Parameters
1	1	1	5,000	0.15	f = 30
2	2	2	6,500	0.16	$x_1 = 2$
3	1	2	5,000	0.15	$y_1 = 3$
4	10	10	10,000	0.16	

Customer i	A_i	B_i	DR_i	OR_i	CR_i	Customer i	A_i	B_i	DR_i	OR_i	CR_i
1	1	6	45,000	100	0.06	11	3	6	15,000	300	0.05
2	2	7	7,500	10	0.07	12	3	7	15,000	250	0.04
3	3	8	7,500	20	0.06	13	4	8	18,000	600	0.05
4	4	9	5,250	10	0.07	14	5	9	4,500	150	0.06
5	5	10	3,750	25	0.05	15	6	10	7,500	100	0.07
6	6	1	3,000	50	0.04	16	6	2	3,000	50	0.06
7	7	2	1,500	40	0.05	17	7	3	3,750	50	0.07
8	8	3	9,000	35	0.06	18	7	5	6,750	50	0.05
9	9	4	9,750	50	0.07	19	9	5	18,000	600	0.06
10	10	5	10,500	100	0.05	20	10	6	22,500	500	0.08

30.6 km (actual distance ≈ 50.4 km). The new warehouse is also supplied by its nearest vendor with a straight-line distance of 28.2 km (actual distance ≈ 39.5 km). The remaining vendors are not assigned to any warehouse because of their higher, weighted, straight-line distances. Customers are divided into two groups (see Table 4); 277 customers are allocated to the old warehouse, while 633 customers are allocated to the new one. Figure 3 depicts the customer allocation for each warehouse. It is observed that a majority of customers is served by the second warehouse because their sites are dispersed in the upper part of Bangkok. Customers in the lower part of Bangkok are served by the old warehouse. Table 4 also provides the product quantities transferred via each warehouse.

Customer Demand Sensitivity Analysis

As customer demands directly affect the warehouse location, the following customer demand sensitivity analysis was conducted. Suppose that the proposed location of the second warehouse is the point of origin. Thus, the area can be divided into four quadrants. Let Q1, Q2, Q3, and Q4 denote quadrant 1, 2, 3, and 4, respectively. Figure 4 shows that all customers can be divided into four groups according to the four quadrants. The following eight scenarios of sensitivity analysis were performed.

Scenario I refers to the case where DR_i for $i \in Q1$ is increased by 100 percent, 200 percent, and 300 percent, while the other parameters are fixed. Note that the 300 percent change is the highest step based on the long-term, marketing expansion plan. Scenarios II, III, and IV are similar to Scenario I, but the increasing parameter is changed to DR_i for $i \in Q2$, Q3, and Q4, respectively. Scenarios V–VIII focus on changes in the opposite direction of customer demand, so that DR_i for $i \in Q1$, Q2, Q3, and Q4 is decreased by 100 percent, 200 percent, and 300 percent, instead. The results are shown in Figures 5 and 6.

From the sensitivity analysis, two conclusions can be drawn.

1) Displacement direction of second warehouse

From Figures 5 and 6, there are three points in each quadrant representing the displacement of the second warehouse. They outline the 300 percent, 200 percent, and 100 percent changes. Changing customer demands always moves the proposed location with the direction depending on whether the demand increases or decreases. When the customer demands increase (Scenarios I–IV), they will draw the warehouse location toward them. For example, when $DR_{i \in Q1}$ increases (Scenario I), the second warehouse will move into Q1 (see Figure 5). Conversely, when the customer demands decrease (Scenarios V–VIII), it will move in the opposite direction. As another example, when $DR_{i \in Q1}$ decreases (Scenario V), the second warehouse will move into Q4, instead (see Figure 6). This shows the effect of demand growth (or of forecasting accuracy). If the company has a marketing expansion plan in a specific area, the second warehouse tends to move into that area. Locating the warehouse far from that area will result in high distribution costs in the long run.

2) Amount of displacement (degree of sensitivity)

The amount of displacement varies according to the magnitude of the change in demand and the area of change.

Table 2
Solution of the numerical example

E_j	F_j	DV_{1j}	DV_{2j}	V_{1j}	V_{2j}	$\beta_{1j}V_{1j}t_{1j}$	$\beta_{2j}V_{2j}t_{2j}$	z
1	1	0	0	2.24	6.85	0.00	0.00	19,146.90
2	2	48,000	0	1.00	5.66	35.45	0.00	
1	2	0	0	1.41	5.89	0.00	0.00	
10	10	0	168,750	10.63	7.57	0.00	612.99	
A_i	B_i	R_{1i}	R_{2i}	s_{1i}	s_{2i}	$\alpha_i R_{1i} s_{1i}$	$\alpha_i R_{2i} s_{2i}$	Other solution
1	6	3.16	2.43	0	1	0.00	1,965.47	$x_2 = 2.8$
2	7	4.00	1.02	0	1	0.00	1,605.26	$y_2 = 7.6$
3	8	5.10	0.44	0	1	0.00	293.95	
4	9	6.32	1.83	0	1	0.00	2,015.04	$\Sigma_i DR_i s_{1i} = 48{,}000$
5	10	7.62	3.24	0	1	0.00	728.78	
6	1	4.47	7.33	1	0	321.99	0.00	$\Sigma_i DR_i s_{2i} = 168{,}750$
7	2	5.10	6.99	1	0	286.82	0.00	
8	3	6.00	6.93	1	0	2,777.14	0.00	
9	4	7.07	7.15	1	0	2,895.60	0.00	
10	5	8.25	7.64	0	1	0.00	1,202.55	
3	6	3.16	1.61	0	1	0.00	120.93	
3	7	4.12	0.63	0	1	0.00	45.24	
4	8	5.39	1.24	0	1	0.00	55.94	
5	9	6.71	2.59	0	1	0.00	139.74	
6	10	8.06	3.98	0	1	0.00	626.99	
6	2	4.12	6.44	1	0	445.30	0.00	
7	3	5.00	6.22	1	0	787.50	0.00	
7	5	5.39	4.92	0	1	0.00	996.76	
9	5	7.28	6.70	1	0	393.13	0.00	
10	6	8.54	7.35	0	1	0.00	794.31	

Table 3
Given parameters used in the experiment

Parameters	Values	Parameters	Values
I	910	CV_j	0.15–0.20
J[a]	4	OR_i	20–5,000
CR_i	0.05–0.10	OV_j	5,000–10,000
f	26.39 baht/liter	M	10,000,000
DR_i	45,000–2,000,000 (for a three-year interval)		
A_i, E_j	Between 100.24878 and 100.85716		
B_i, F_j	Between 13.52369 and 14.12159		

[a] Vendor sites are randomly generated

Table 4
Solutions for the case study from the numerical experiment

List	Values
z-Value	29,931.6
Longitude of second warehouse, x_2	N13.80753
Latitude of second warehouse, y_2	E100.56386
Number of vendor assigned to first warehouse	1
Number of vendor assigned to second warehouse	1
Number of customers assigned to first warehouse	277
Number of customers assigned to second warehouse	633
Overall demand	91,742,566
Overall demand fulfilled via first warehouse	27,824,491
Overall demand fulfilled via second warehouse	63,918,075

A small amount of displacement means a low impact on the decision and vice versa. Figures 5 and 6 show the amount of displacement in accordance with changing levels of demand and areas of change. It was found that higher sensitivity occurred in Scenarios I and II (≥3.0 km), in areas opposite to the first warehouse with reference to the point of origin. The remaining scenarios showed lower sensitivity (≤3.0 km). Note that the 3-km breaking point was defined by top management. This finding allows the manager to focus on increasing customer demands in Q1 and Q2. A greater increase in demand in these areas can result in a large warehouse displacement. However, increased demand in Q3 and Q4 has little effect because the customer demands served by the first warehouse have no effect on the location displacement of the second warehouse.

To sum up, for top management, we recommend making a decision for this particular scenario as follows. The proposed location of the second warehouse is on the northeast side of the first warehouse (N13.80753, E100.56386) with a straight-line distance of 19.5 km. The accuracy of forecast demand or growth has a significant effect on the proposed location. An area with increasing demands will attract the warehouse location, while an area with decreasing demands will repulse the warehouse location. The amount of displacement depends on the level of the change in demand and the change points. Suppose that the proposed location is a point of origin. One important point that top management should pay particular attention to is increasing demands in the upper distribution area (opposite the first warehouse), because this has a great impact on location displacement (maximum of 7.4 km). On the other hand, increasing demands in the lower distribution area (close to the first warehouse) or decreasing demands in all areas have less significance (based on this scenario analysis) because they have little effect on location displacement (within ≈3 km of the proposed location). This recommendation can be useful in the selection of the location of the new warehouse in relation to the company's marketing plan in the future.

Figure 2 Location of first and second warehouses

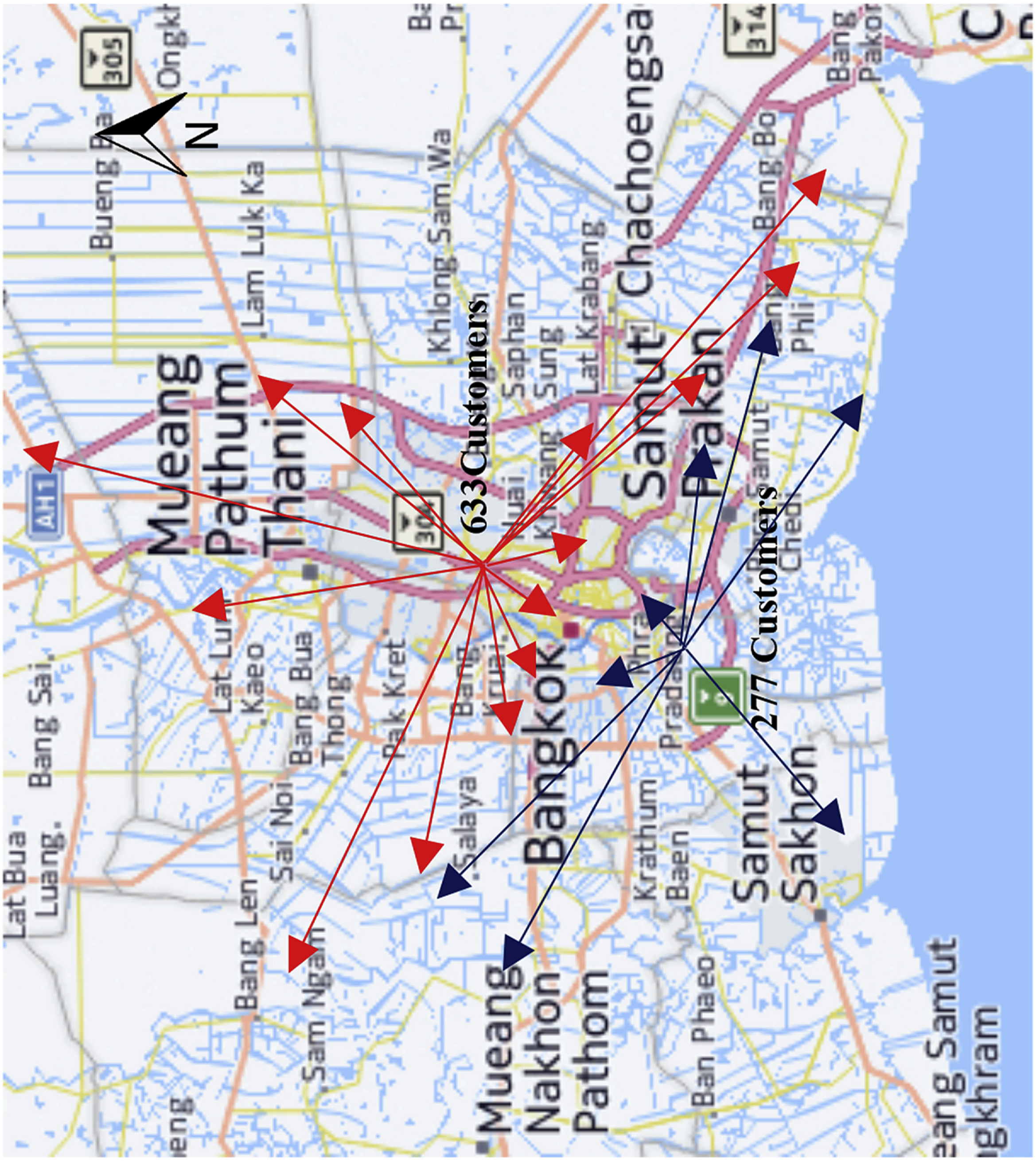

Figure 3 Outline of customer allocation

Figure 4 Four quadrants for sensitivity analysis

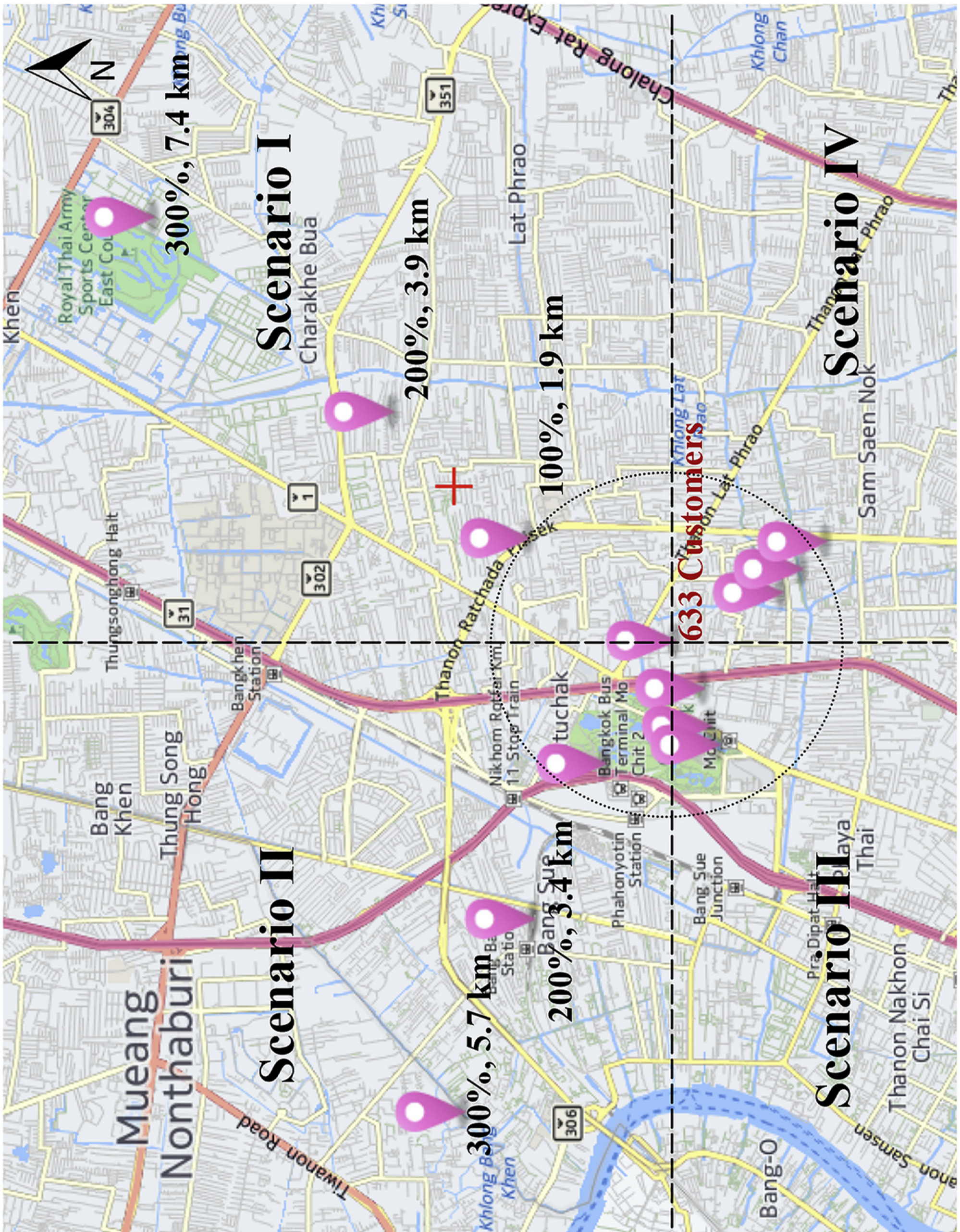

Figure 5 Location displacement of second warehouse due to Scenarios I–IV

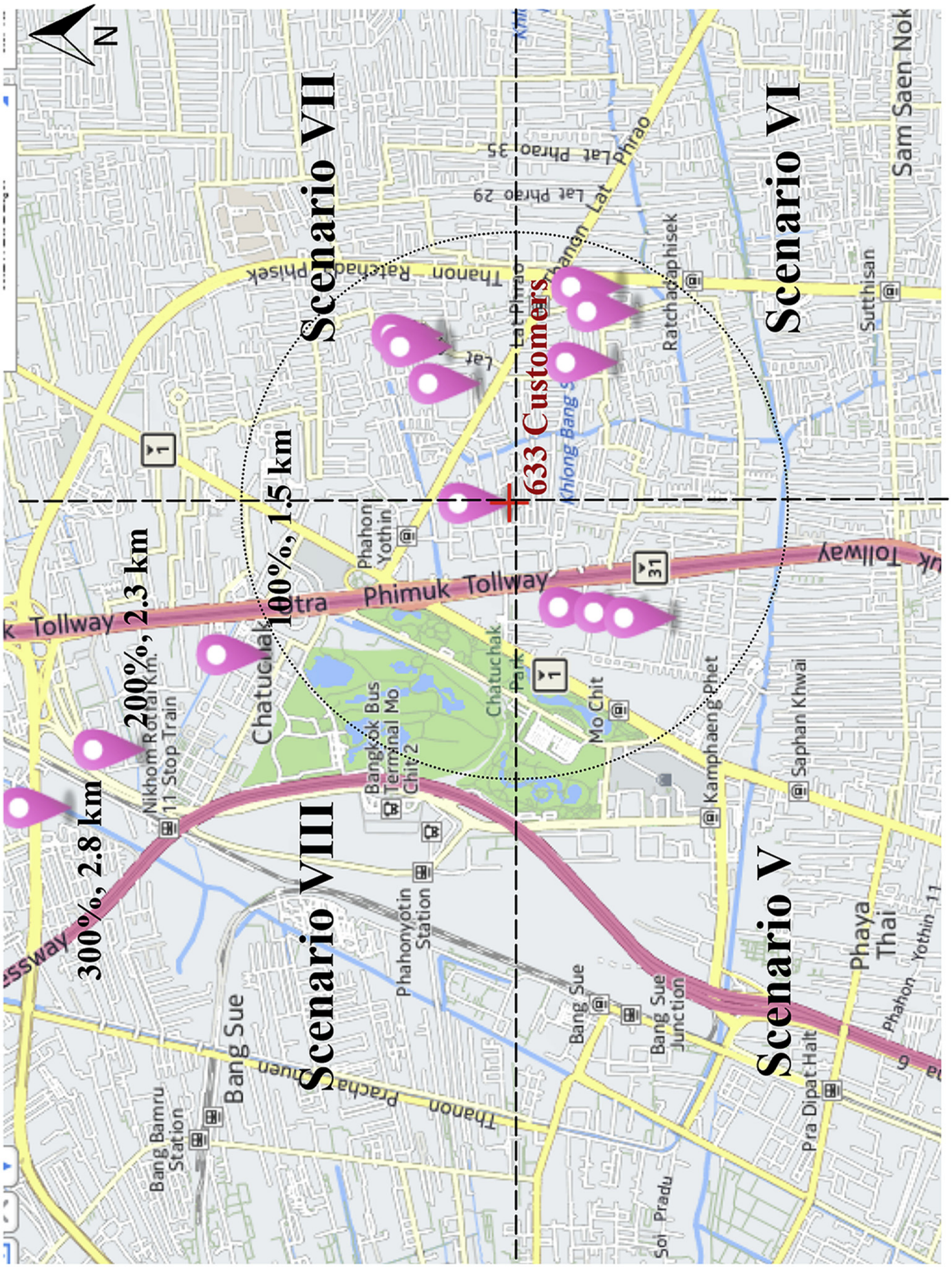

Figure 6 Location displacement of second warehouse due to Scenarios V–VIII

Conclusion

This paper investigated a particular problem of a paper wholesaler in Bangkok, which was first reported in Monthatipkul (2012). Since the company is facing high transportation costs and the first warehouse is located in an unsuitable place, the managing director has decided to open a new warehouse, to save on transportation costs in the long run. Unfortunately, the model of Monthatipkul (2012) ignored transport costs from the supply side, which can affect the location of the new warehouse. This paper strengthens the model of Monthatipkul's (2012) by including transportation costs from the supply side.

This paper proposed a non-linear model using the straight-line distance for finding an appropriate, approximate location of the second warehouse. By solving the model, we can determine the location of the second warehouse and inform the customer/vendor allocation decision. Some customers/vendors must be re-assigned to the new warehouse, while the remainder will stay assigned to the old warehouse. Sensitivity analysis showed the directions and amounts of displacement of the second warehouse location. It was observed that changing overall demands in each quadrant always moved the warehouse location and the displacement direction varied according to increasing/decreasing overall demands, which always attracted/repulsed the warehouse location. The amount of displacement depended on customer sites and the amount of change. This insight can provide useful information in the decision making undertaken by top management.

In general, this article provides guidelines for determining the location of a second warehouse for a company. However, as the decision to establish the second warehouse is very important, a series of additional tasks should be performed. Future study should address the following tasks: 1) intensive actual surveys and studies of the land, neighborhood, infrastructure, traffic conditions, workforce, laws, among others; and 2) a feasibility study of the suggested location, including a financial feasibility study.

Conflict of interest

There is no conflict of interest.

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