



The investigation of interpretation shift among elementary students in semiotic reasoning for constructing a concept in math

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Article Info

Article history:

Received 7 September 2022

Revised 8 January 2023

Accepted 10 January 2023

Available online 17 November 2023

Keywords:

construction concept,
semiotic reasoning,
shift in interpretation

Abstract

Semiotic reasoning believed as a model of learning leads students to be able to have multiple interpretations of a certain object or sign. This study aimed to describe the shift in the interpretation of fourth-grade elementary school students in semiotic reasoning in constructing rectangular concepts. The procedure of this research was exploratory descriptive. The subjects in this study were three fourth-grade students in private elementary schools in Jember who performed semiotic reasoning and experienced a shift in interpretation. This shift in the interpretation occurred at the stage of identifying objects. When working individually, the subjects considered that folded paper was rectangular. After discussing in a group, the interpretation of the subject shifted. The subjects found that the folded paper was not rectangular. This shift in the interpretation also occurred at the sign-making stage. Images made by the subject were rectangular. Therefore, the subject made a rectangular image according to the object that has been identified. When working individually, the subject was not able to identify the characteristics of rectangular objects carefully. Therefore, the characteristics of rectangular-shaped objects were found inappropriate. After discussing it with friends in the group, the subject's interpretation underwent a shift. The interpretation shifts were in object identification, making signs and making meanings. The shift that occurred at the stage of making meaning resulted in a change in the students' interpretation of in-setting concepts.

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Introduction

Reasoning is crucial in mathematics learning. By reasoning, students can draw conclusions based on facts that have been proven true. It can be defined as an

activity to conclude facts, analyze data, estimate, explain, and make a conclusion. (Panchal, 2013) defined reasoning as the process of reaching logical conclusions based on relevant facts and sources. The reasoning process in concluding is an activity that requires high-level thinking skills. In this study, the reasoning is defined as the ability to think logically, which is used in solving problems or tasks to obtain new conclusions or statements based on facts whose truth has been proven or assumed previously.

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Semiotics is the study of signs. In learning mathematics, children first learn exclusively with signs. Mathematical activities are performed by interpreting and transforming signs. Learning mathematics does not only involve taking the meaning of the conversion of mathematical signs but also depends on various possible interpretations. Semiotics in mathematics is defined as the use of symbols that are very helpful in understanding the process of thinking, symbolizing, and communicating (Ostler, 2011; Presmeg et al., 2008). Semiotics in mathematics teaching usually refers to the teaching of mathematics as a process of manipulating symbols through structured algorithms and strictly defined in theorems.

As for students, mathematics learning activities can involve interpreting and transforming signs to develop mathematical knowledge (Stjernfelt, 2015). In making a sign, it matters to know how meaning emerges from a sign when it is used to communicate (Suryaningrum et al., 2018). In learning mathematics, a person needs signs and representations. Signs are used to think about mathematical relationships with real-world objects. Therefore, signs are the result of thinking that is used to produce other new signs. A sign is a representation of an object. The interpreter is a thought or notation to represent an object. Thus, each sign can act as an object or as an interpreter of other signs (Kralemann et al., 2013; Minarni et al., 2016).

Semiotic reasoning is reasoning related to signs (Suryaningrum et al., 2020b). It is the activity of drawing conclusions based on the objects that have been identified, the sign made based on the object, and the interpretation of a sign. Semiotic reasoning in constructing concepts is the process of drawing conclusions based on objects, signs, and interpretations of signs. Constructing concepts is a particular way in which students try to understand available information, gather facts, conduct investigations, and make connections using the entire cognitive structure to form concepts. In this research, semiotic reasoning in constructing concepts is an activity of concluding stages of identifying objects, making signs, defining, revising concepts, and forming concepts. In this research, semiotics refers to Peirce's theory.

Research on semiotics has been carried out by several researchers including the results of Schreiber's research (2013) reporting that when elementary school students are given the number 8060, there are two different interpretations. Ng and Sinclair (2015) investigated children's learning about reflectional symmetry in dynamic geometry environment. The results of Dimmel and Herbst's (2015) study stated that there is a semiotic list of geometric diagrams. Research conducted by

Suryaningrum and Ningtyas (2019) reported that after the three research subjects understood the problem, three subjects found five different objects, made a new sign in the form of a picture, and solved one problem resulting in three different interpretations. Of the several studies conducted by the researcher, no one examined the shift in students' interpretations of semiotic reasoning. This study aims to describe the shift in the interpretation of fourth-grade elementary school students in semiotic reasoning in constructing rectangular concepts. This research is important to do so that teachers can be used as a learning model that can lead to multiple interpretations of students in learning.

Theoretical Framework: Peirce's Semiotic Theory

Stated is that semiotics is identical to the concept of logic that focuses on knowledge of the human thought process (Peirce, 1958). Peirce's ideas about signs, use logic and metaphysics and offer a more comprehensive theoretical framework for a cultural context called social semiotics (Turkcan, 2013). According to Peirce, someone thinks through signs, enabling them to communicate with each other and give whatever meaning is in their environment. Peirce's semiotic theory better integrates individual interpretations and gives freedom of interpretation (Suryaningrum & Ningtyas, 2019). The basic principle of Peirce's theory is that everything can be a sign, as long as it can represent something according to individual interpretations and thoughts (Sendera et al., 2014). A sign is a representation of an object. Interpretation is a thought or notation to represent an object. Each sign can act as an object or as an interpreter of other signs (Eco, 1981; Schreiber, 2013).

According to Peirce, thinking using signs enables students to communicate with other friends and give meaning to whatever they identify in their environment (Sendera et al., 2014; Tarasenkova & Kovalenko, 2015). The basic principle of Peirce's theory is that information obtained from the environment can be a sign, provided that students can interpret something that has been identified and thought (Stjernfelt, 2015; West, 2015). A sign can evoke an interpretant which is another sign that is equivalent to what is in someone's mind (Stables & Semetsky, 2015). Peirce's theory of signs focuses on three dimensions or triads that divide the sign into three parts, namely, objects, signs, and interpretants (Kralemann et al., 2013; Metro-Roland, 2009; Murphy & Ornsten, 1976; Presmeg et al., 2008; Schreiber, 2013; Sherzer, 2009; Yang & Hsu, 2015). The triadic relationship of Peirce's theory can be described in [Figure 1](#) as follows.

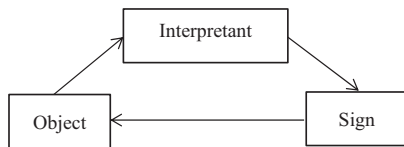


Figure 1 Triadic relationship of signs based on Peirce's theory

A sign is something in the physical form that can be captured by the five human senses and is something that refers to (represents) something other than the sign itself (Eco, 2018; Kralemann et al., 2013; Sendera et al., 2014). The sign acts as a mediator between the object with the knowledge and meaning of the sign (Brier, 2018). Signs can be interpreted in several different ways (Suryaningrum, 2018). An object is also referred to as a reference. The object or sign reference is a social context that becomes a reference to the sign or something the sign refers to (Eco, 1984; Schreiber, 2013). An object is something that represents the interpretant produced (Godzich et al., 1978). The object referred to by a sign is reality or whatever is thought to exist. This means that an object does not have to be concrete, it does not have to be something that can be seen by the eye or as an empirical relationship but can also be other abstract entities. One object can be represented by different signs (Suryaningrum & Ningtyas, 2019). Interpretation or use of signs is the concept of thought of people who use the sign and reduce it to a certain meaning that is in someone's mind about the object to which a sign is referred (Sarbo & Yang, 2015; Yang & Hsu, 2015). The most important thing in the semiotic process is how meaning arises from a sign when it is used by people when communicating. Interpretation is another representation that is referred to for the same object (Godzich et al., 1978). The Semiotic theory states that learning should emphasize freedom of interpretation (Peirce, 1958). One's mind will grow interpretations of other people's activities related to the object. Interpretation is a response to the object through the mediation of the sign. Interpretation can also be referred to as the reaction of signs, feelings, and thoughts, which are referred to as sign meanings.

Methodology

Design

This research used the descriptive explorative method to investigate the shift in students' interpretation of semiotic reasoning by students in learning rectangular. Semiotic reasoning meant here was a conclusion making based on an object identified, signs made by students as well as how students use interpretation to make meaning of an object made, where in the end, a formula will be found (Creswell, 2012). In addition to using Creswell's analysis, the researcher analyzed the results of tasks done by the students using Peirce's semiotic theory (Colapietro, 1987; Peirce, 1958), where semiotics is divided into three parts, namely, object, signs, and interpret. The semiotic reasoning framework based on Peirce's semiotic theory is presented in Table 1 below.

Source of Data

The data in this study were: (1) audio-visual recording data, when the learning process was used to record student activities in constructing rectangular concepts; (2) student record data in learning, when constructing a rectangular concept was used to obtain an overview of the object identified, signs made by students based on the object, and student's interpretation in constructing the concept of a plane shape; therefore, when working individually or in groups, they would find shifts in students' interpretations when constructing a rectangular concept; (3) observation data were used to observe students while working on a worksheet; and (4) the interview data were used to explore shifts in the interpretation of students in semiotic reasoning in constructing the concept of a plane figure and completing the data if the data from the learning video and student notes are unclear. Interviews were carried out after the learning process activities for three research subjects for approximately thirty minutes.

Table 1 The Semiotic reasoning framework based on Peirce's theory

Dimension of Peirce's theory	Semiotic Reasoning Components	Student Activity
Object	Object identification	Students identify rectangular objects in their environment
Sign	Making signs	Students make signs based on objects that have been identified
Interpret	Make meaning	Students mention the properties of the sign
	Form a concept	Linking the properties of signs with prior knowledge to conclude concepts
	Revise concepts	Look again at the stages of concept construction
	Establish the concept	Establish the concepts found

Participants

The subjects in this study were three fourth-grade students in private elementary schools in Jember Regency. The procedures for selecting research subjects were: (1) Conditioning, which was determining the location or place to find prospective research subjects; (2) Observing the learning process to find out semiotic reasoning; (3) based on observations, choosing six students who experienced a shift in the interpretation of semiotic reasoning; and (4) Subject selection using purposive sampling from three of six students, who were taken to be the subjects of research with the subject's criteria being able to communicate its reasoning both verbally and in writing.

Procedures

This research began with recording the learning process carried out by the teacher. The researcher recorded all student activities when supervising learning on the rectangular material. This activity was carried out to observe student activities when constructing the concept of rectangular. After following the learning process, students were asked to collect student notes when constructing concepts. Student notes were studied to identify shifts in student interpretation when constructing the rectangular concept. After collecting student notes, the researcher conducted interviews with the research subjects. Interviews were conducted to explore information that has not been obtained from the recording of the learning process and the results of students' notes.

Data Analysis

Data were analyzed to describe the shift in the interpretation of students' semiotic reasoning in constructing rectangular concepts. Data analysis was carried out through three stages, namely: (1) changing verbal data that had been collected into written data to be analyzed; (2) selecting data in accordance with the focus of the study; (3) presenting data by grouping data based on Peirce's semiotic theory consisting of three elements, namely, objects, signs, and interpretants (Godzich et al., 1978); (4) analyzing the shift of interpretation that occurs in semiotic reasoning; (5) drawing conclusions based on research findings data; and (6) triangulating data to determine whether the data were saturated (Creswell, 2012).

Results

This study describes the shift in the interpretation of fourth-grade elementary school students in Jember Regency, analyzed by three students who experienced a shift in interpretation in constructing the rectangular concept.

First Subjects

The first subject (S1) is a subject that experiences a shift in interpretation when making meaning. In the process of learning rectangular material, in the initial stage, the teacher asked students to collect rectangular objects that students have seen. The rectangular objects found by S1 were pencil cases, blackboards, and TV screens (object identification). The next activity was to make pictures of these objects on the paper prepared by the teacher (making signs). Images of rectangular objects made by S1 can be seen in Figure 2 as follows.

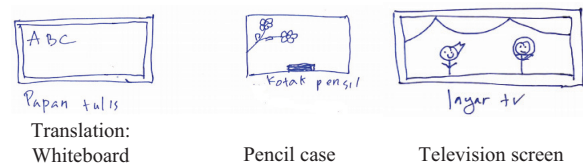


Figure 2 Rectangular objects made by S1

S1 found the characteristics of each rectangular object (make meaning). S1 wrote a blackboard's characteristics i.e. the sides were not the same length, and there were four sides. The characteristics of the pencil case found by S1 were four sides and four angles while the characteristics of the television screen found by S1 were that the sides were not the same length and with four angles. After discovering the characteristics of objects, S1 discussed the results of their work with friends in the same group. After discussing with the group, S1's interpretation of the rectangular shape object changed. In this case, S1 experienced a shift in interpretation in his mind in interpreting the sign. The characteristics of each object were rectangular (make meaning). After group work by S1, there was a change. S1 wrote the characteristics of a blackboard were that the sides were not the same length, there were four sides. The characteristics of the pencil case found by S1 were four sides and four angles while the characteristics of the television screen found by S1 were that the sides were not the same in length and with four angles.

The activity of finding the properties of a rectangle is called form the concept. From the characteristics of three rectangular shapes, S1 wrote the features of a rectangle with four sides and four angles. After finding out the properties of the rectangle, S1 looked back at the properties that had been found (revising the concept). After assuming the properties of the rectangle written were correct, S1 determined the properties of the rectangle to have four sides and four angles (determine the concept). The shift in interpretation by S1 in constructing a rectangular concept can be seen in the following Figure 3.

Second Subject

The second subject (S2) is a subject that experiences a shift in interpretation when making meaning and forming concepts. The rectangular objects found by S2 were paper, textbooks, and blackboards (object identification). After collecting rectangular objects, S2 drew pictures of the objects on the paper prepared by the teacher (making signs). Images of rectangular objects made by S2 can be seen in Figure 4 as follows.

After making a picture, S2 wrote the characteristics of a paper object with an angle 90° , and the sides were not the same. The characteristics of the blackboard were an angle of 90° and were not equal. The characteristics of a package book were an angle of 90° and the left side was the same as the right side. They wrote the characteristics of individual rectangular objects and discussed the results of their work with friends in the group. In these activities, there was a shift in interpretation in S2's mind in

interpreting the sign (making meaning). S2 wrote the characteristics of paper and textbooks were an angle of 90° and the two sides were the same length whereas the features of the chalkboard were 90° and the two sides were called p and l .

In the activity of forming concepts, the interpretation of S2 changed. After the group discussion, S2's interpretation changed. S2 wrote the properties of a rectangle were 90° and the two sides were the same length. From the characteristics of three rectangular objects, S2 wrote the features of a rectangle that was at an angle of 90° and the two sides were the same lengths. After finding out the properties of the rectangle, S2 looked back at the properties of the rectangle that had been found (revising the concept). After assuming the properties of the rectangle written were correct, S2 specified the properties of the rectangle were at an angle of 90° and the two sides were the same length (determine the concept). The shift in the interpretation of S2 in constructing rectangular concepts can be seen in Figure 5.

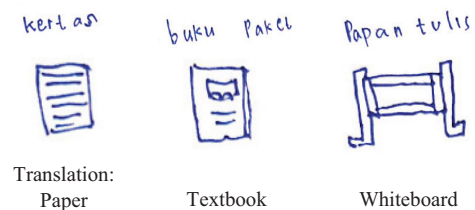


Figure 4 Rectangular objects made by S2

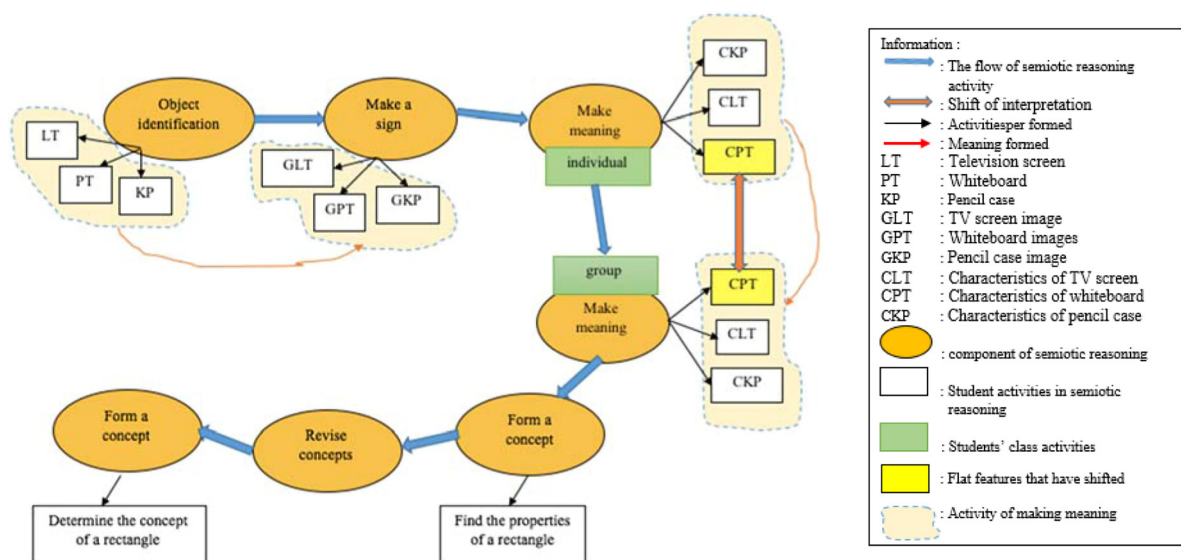


Figure 3 Interpretation shift in S1 Semiotic reasoning in constructing rectangular concept

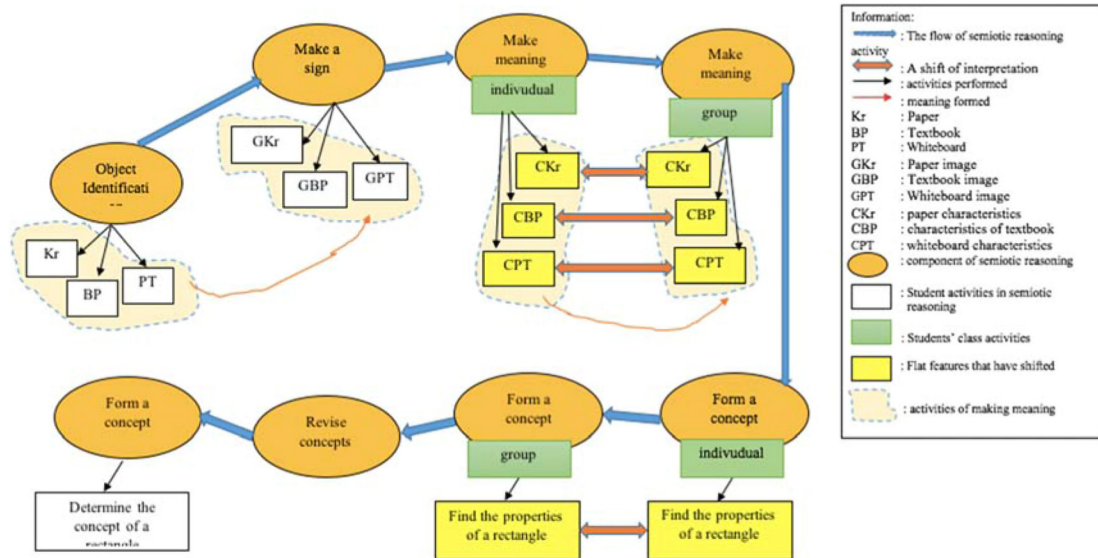


Figure 5 Interpretation shift in S2 semiotic reasoning in constructing rectangular concepts

Third Subject

The third Subject (S3) was a subject that experiences a shift in interpretation when object identification, making meaning, and forming concepts. In the first stage, S3 collected rectangular objects in the classroom. Rectangular objects found by the students were television screens, folded paper, and desks (object identification). After collecting rectangular objects, S3 drew these objects on paper that had been prepared by the teacher (making signs). Images of rectangular objects made by S3 can be seen in Figure 6 as follows.

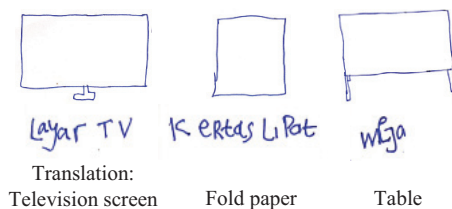


Figure 6 Rectangular Objects Made by S3

After drawing, S3 discovered the characteristics of each rectangular object (making meaning). S3 wrote the characteristics of a television screen with the same length of two sides, and two angles in the same width. The characteristics of folded paper found by S3 were two sides at the same length, two corners were the same width. The characteristics of the table found by S3 were two sides at the same length, and two corners were the same width. After writing individually the characteristics

of square objects, S3 discussed the results of his work with friends in the same group. In the group discussion, S3 discussed the rectangular objects identified. From the results of group discussions, S3's interpretation changed (there was a shift in interpretation). S3 found that the surface of the folded paper was not rectangular, but square. Realizing this, S3 identified another rectangular object he had seen. S3 found the blackboard was a rectangular object. The next activity carried out by S3 was to discuss the characteristics of the rectangular shape discovered. From the results of discussions with the group, S3's interpretation of rectangular objects (signs) changed. In this case, there was a shift in interpretation in S3's mind in interpreting the sign. The characteristics of each object were rectangular (making meaning) after the group work found by S3 and then changed. S3 wrote the characteristics of a television screen were two sides at the same length and two angles of 90° . The characteristics of the blackboard found by S3 were two sides at the same length and two angles of 90° . The characteristics of the table written by S3 were 2 sides at the same length and two angles were 90° .

After discovering the characteristics of rectangular-shaped objects, S3 connected these characteristics with material that had been studied previously, namely, the properties of rectangles (linking signs with prior knowledge). This activity of discovering the properties of a rectangle is called forming a concept. In the activity of forming concepts, the interpretation of S3 changed. Before discussing with his groups, S3 wrote that the properties of a rectangle were two sides at the same

length and two angles at equal width. After the group discussion, S3's interpretation changed. S3 wrote that the properties of a rectangle were two sides at the same length and two angles were 90° .

From the characteristics of three rectangular objects, S3 wrote the features of a rectangle, that it had two sides at equal length and two angles of 90° . After finding out the properties of the rectangle, S3 looked back on the properties of the rectangle that had been found (revise a concept). After assuming the properties of the rectangle were written correctly, S3 specified the properties of a rectangle were to have two sides of equal length and two angles of 90° (determine the concept).

In the activity of identifying objects, there was a shift in interpretation. Before discussing with the group, S3 found three objects considered rectangular. However, after discussing it with the group, there was a shift in the interpretation by S3. The shift occurred in a folded paper object. Initially, S3 considered the surface of the folded paper as rectangular. After discussing the matter with the group, there was a change in interpretation. S3 found that the shape of the folded paper was square. After realizing that the folded paper was square, S3 searched for a rectangular object again. S3 found the blackboard was a rectangular object. The shift in the interpretation of S3 in constructing rectangular concepts can be seen in the following Figure 7.

Discussion

In the activity of identifying objects, subjects collect rectangular objects around them. These objects are objects collected by students from observations of signs that are on the student's environment (Schreiber, 2013; Suryaningrum et al., 2020a; Suryaningrum et al., 2020b). In the activity of identifying objects, there are different objects collected by this subject influenced by the subject's experience in seeing rectangular-shaped objects. This opinion is in accordance with the opinion (Schreiber, 2013) stating that each individual creates an interpretation with the background of his experience. The objects identified by the subject also depend on the interpretation of the subject in identifying objects around them. This is consistent with the opinions (Brier, 2015; Sendera et al., 2014) which stated that the objects observed are objects that represent interpretations.

In identifying objects, S1 experienced a shift in interpretation. When identifying objects individually, the subject considered folded paper to be rectangular. After the group discussion, the subject's interpretation changed. The subject found that the folded paper was not square, but square. This activity is in accordance with the opinions of (Kim et al., 2013) who stated that through interaction with peers, students will look back on their

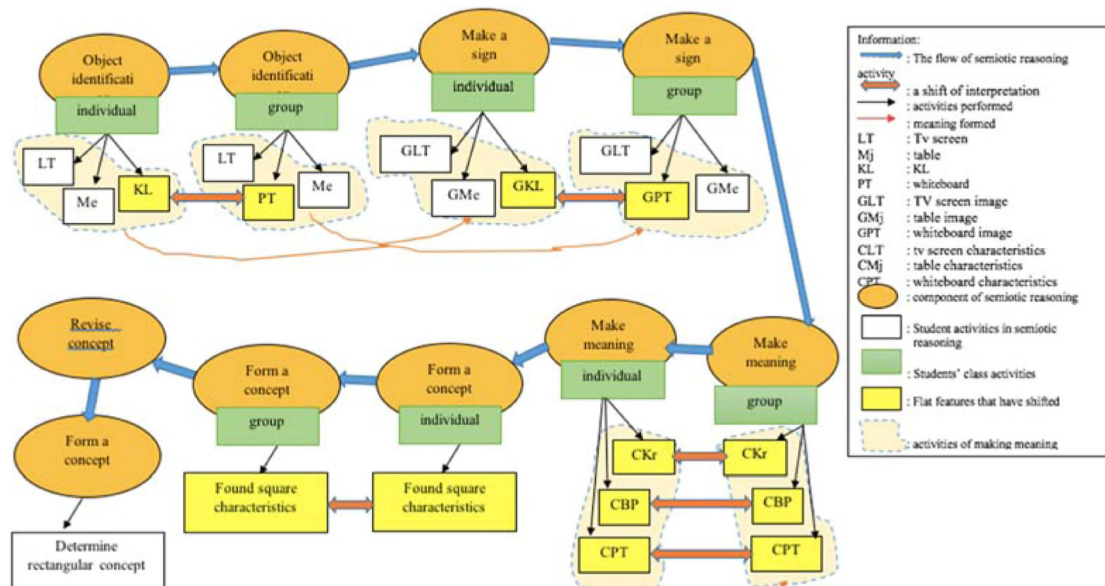


Figure 7 Interpretation Shift in S3 Semiotic Reasoning in Constructing Rectangular Concepts

previous work to improve students' knowledge and understanding, and revise and correct misunderstandings. The activities carried out by S1 showed that after discussing with friends in the group, S1's interpretation changed. This is consistent with the opinion (Hastuti et al., 2016) which stated that small group discussion activities influence group members to re-examine the results of their work and revise the results of the initial work of students.

This shift in interpretation also occurred at the sign-making stage. Activities that were carried out by the subject at the stage of making a sign were drawing pictures. Formation of signs (sign) are carried out for the benefit of communication. One can represent mathematical ideas through symbols or signs (Suryaningrum et al., 2018). According to Peirce (1958), a person thinks through signs, enabling them to communicate with each other and give whatever meaning is in their environment. When identifying objects, the three subjects found the same object, the blackboard, but in making a mark, the three subjects made a different picture. This is consistent with the opinion (Kralemann et al., 2013) which stated that every interpretation of something as a sign has been dependent on subjective judgments. Thus, it might be possible to act as a different sign for the same object.

When working individually, the subject made a mark in the form of a folded paper image. The image created by the subject corresponded to the object that was identified in the previous stage. Students made pictures to represent their interpretations when looking at objects. This is in accordance with the basic principle of Peirce's theory which stated that everything can be a sign, as long as it can represent something based on students' interpretations and thoughts (Rezaie & Gooya, 2011). A person may have different interpretations related to images; this depends on how the person interprets a picture (Ali & Aslaadi, 2016; Burgos & Godino, 2020).

When discussing with respective groups, the subject underwent a shift in interpretation, namely, the subject found that the surface of the folded paper was not rectangular, but rather square. The findings of this study are in accordance with one of the social-based characterizations of interpreting various perspectives (Magiera & Zawojewski, 2011), which implied that one's thoughts are influenced by mathematical communication with others by considering new information obtained from peers who together understand the concept of mathematics. After making a picture (making a sign), the activities carried out by the subject are interpreting the sign. The activity of interpreting the sign is an activity of finding the characteristics of the rectangular-shaped

objects that have been identified. In the activities of interpreting the sign, the subject uses its interpretation to find the characteristics of each object.

In the activity of making meaning, the activity carried out by the subject was to use interpretation to interpret the sign to find the concept of a rectangle. Interpretation is a thought or notation to represent an object (Suryaningrum & Ningtyas, 2019). In this activity, the subject interpreted the sign by investigating the sign. The subject counted the number of sides, and the number of angles, compared the size of the sides and found the size of the angle. The activity is in line with the opinion of (Stjernfelt, 2015) which stated that making meaning is an activity of interpreting signs associated with external perceptions of objects that have been observed. The properties of the rectangles found by the three research subjects were different, some were the same. This is in accordance with the opinion of (Ali & Aslaadi, 2016) which stated that someone's interpretation is different related to an image.

When working individually, subjects were not able to identify the characteristics of rectangular objects carefully, therefore, the characteristics of rectangular objects found were incorrect. After discussing it with members of their group, the subject's interpretation underwent a shift. By discussing with friends in the group, the subject knew how to identify the characteristics of rectangular objects by counting the number of sides, measuring each side, comparing the measurement results of each side, counting the number of angles, and measuring the angle. In this way, research subjects can find the characteristics of each object precisely. In this activity, there was a shift in interpretation. From the above activities, it was found that in constructing concepts, students need help from others, both peers and teachers so that the concepts found are valid. This is in accordance with the opinion of (Rajotte, 2015) who stated that, in primary school, teacher learning is expected to help students to analyze their practice, thus, impacting students' pedagogical abilities. The shift that occurs at the stage of making meaning results in a change in the interpretation of students in setting concepts. This was seen after S2 and S3 discussed it with group members, and the interpretation of setting concepts changed.

Conclusion and Recommendation

This shift in interpretation occurs at the stage of identifying objects. When working individually, the subject considers folded paper to be rectangular. After group discussion, the interpretation of the subject shifts.

The subject found that folded paper is not rectangular, but square. After a shift in interpretation, the subject identifies the object again to replace the folded paper that is not rectangular. The subject identifies the objects around them. The subject found a rectangular object, a blackboard. By comparing the shape of the blackboard with the shape of other objects that have been found, the subject is convinced that a blackboard is a rectangular-shaped object.

This shift in interpretation also occurs at the sign-making stage. Activities that are carried out by the subject at the stage of making a sign is to make a picture. The image created by the subject corresponds to the object identified in the previous stage. When working individually, the subject makes a mark in the form of a folded paper image. When discussing with their groups, the subject has undergone a shift in interpretation, namely, the subject has found that the surface of the folded paper is not rectangular, but rather square. The subject replaced the folded paper image with the drawing of the blackboard, which is a rectangular shaped object.

When working individually, the subject has not been able to identify the characteristics of rectangular objects carefully, thus, the characteristics of rectangular-shaped objects found there are inappropriate. After discussing it with friends in the group, the subject's interpretation undergoes a shift. By discussing with friends in the group, the subject knows how to identify the characteristics of rectangular objects by counting the number of sides, measuring each side, comparing the measurement results of each side, counting the number of angles, measuring the angle. In this way, research subjects can find the characteristics of each object precisely. The shift that occurs at the stage of making meaning results in a change in the interpretation of students in setting concepts.

Despite the research findings, there is a research problem that has not been answered. Further research needs to be done to find out the cause of the shift in interpretation.

Conflict of Interest

The authors declare that there is no conflict of interest.

Fundings

This study was funded by the Center for Publication and Scientific Dissemination Management, the University of Muhammadiyah Jember (the number is still in process).

Acknowledgments

Our gratitude goes to the fourth-grade teacher of a private elementary school in Jember Indonesia who helped us in researching so that we could obtain relevant data. We also express our gratitude for the research funding support provided by the head of the research and community service institution, Muhammadiyah University of Jember.

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